To the grand philosophical question: “What is man?”
Aristotle answered:
“Man is a rational animal.”
Modeling Theory offers a new answer:
“Man is a modeling animal!”
Homo modelus!

see Hestenes, p. 34
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Groupe Internationale de Recherche sur l’Enseignement de la Physique (GIREP)
AMSTEL Institute, Faculty of Science, Universiteit van Amsterdam

With the support of:
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Preface

This book and CD contain almost all papers presented at the 2006 Amsterdam GIREP Conference:

The review process for papers was as follows: The chairperson of each paper session conducted a first review and recommended whether or not a paper should be published in the CD proceedings on the basis of relevance and quality and whether or not the paper should be considered for the book. A second review was conducted by reviewers after the conference. Papers receiving two positive book recommendations were accepted for the book version. Papers with one positive book recommendation went through a third review by the editors. Poster papers were reviewed by one reviewer.

All papers in the final version were posted on the web and authors were asked to check for major errors which might have slipped in somewhere in the word processing towards the final format.

All book papers have also been included on the CD. The CD also includes the photo collection of the conference.

This book and CD constitute one product with one ISBN number. When referring to any paper, whether in the book or on the CD, the reference should be:

[Author name](2008).[paper title]. : E. van den Berg, A.L. Ellermeijer, O. Slooten (Eds.), Modelling in Physics and Physics Education, [page number]. Amsterdam: AMSTEL Institute, University of Amsterdam.
Interpreting Diffraction Using the Quantum Model

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Abstract
In previous researches we designed and implemented an educational path to construct the theoretical quantum mechanical model, following the Dirac vectorial outline, in the secondary school. In analysing the phenomenon of polarisation students are introduced to quantum concepts and construct their new ideas about: the peculiar concept of state; the superposition of states; the meaning of incompatible observables; the basic formalism of vectorial space. Interpreting diffraction, within the conceptual framework of our proposal, constitutes simultaneously an extension, a potentiality and a strengthening of the proposal itself.

In the context of the Italia-Slovenia Interreg III Project, we designed a didactical model for teachers in order to interpret diffraction patterns. The development of this model starts from the quantum model of polarization constructed above. The diffraction model is based on the identification of mutually exclusive potentialities of photon transmission through a single slit. The pattern derived from the diffraction model is in good agreement with the experimental one, in the Fraunhofer approximation.

Introduction
In the panorama of proposals concerning the teaching of quantum mechanics in secondary schools (Phys. Educ., 2000; Am. J. Phys, 2002) we may identify a stream that adopts the strategy of analysing specific phenomenologies to construct quantum concepts. This makes reference to the Dirac’s vectorial description of quantum states (Dirac, 1958; Sakurai, 1985) and hinges upon the discussion of the principle of quantum linear superposition, as a founding principal of the new theory, and also pays attention to the role played by formalism in attributing meaning to physical entities (Feynman, 1965; French, 1975; Toraldo di Francia, 1975; Ghirardi et al., 1995; Pospiech, 2000; Holbrow et al., 2002).

Our proposal concerning the teaching of quantum mechanics is linked to this stream (Ghirardi et al. 1997; Michelini et al., 2000, 2001). The phenomenology of optic polarisation constitutes a privileged context for the constructing of a bridge from classical to quantum physics for successive levels of conceptualisation from the phenomenological laws studied in the laboratory, to their analysis in an ideal single photon context, to their discussion for the construction of the concept of state and of superposition, and to the mathematical formalisation at the base of interpretation (Cobal et al., 2002; Michelini et al., 2002; Michelini, Stefanel, 2004).

The research conducted on-site have demonstrated the effectiveness at secondary school level of a strategy based upon the development of logical arguments by the students in a coherent exploration of different interpretative hypotheses, constructed by students itself in the context of our educational path. The results have shown that the students grow more competent in the quantum descriptions of phenomena through the concept of state, with a sufficient mastering of the basic formalism and an understanding of its conceptual role (Michelini et al., 2001; 2004).

The use of concepts in diverse contexts consolidates the learning process, while extending its depth. We therefore pose the objective of building a model for the quantum description of optic diffraction without relying upon classical interpretation. The choice is dictated by the cultural and applicative relevance of such a phenomenology (Bohr, 1961), and by the availability of on-line sensors, which facilitate its exploration and provide the opportunity for facing an interpretation not limited to the case of bi-dimensional vectorial spaces.

In the context of the Italia-Slovenia Interreg III Project and Italian national Project PRIN-Fis21, we designed a didactical model in order to interpret diffraction patterns. Below we present the case of a single slit diffraction.

The phenomenology of the single slit diffraction
With two on-line sensors, one of position and one of light intensity, we may acquire the distribution of light intensity produced by a single slit diffraction process, as illustrated in Figure 1 (Corni et al., 1993). Analysing the distribution as a function of position we recognize that:
D1) $I_{\text{max}} \propto 1/D^2$; the maximum value of the distribution $I_{\text{max}}$ is proportional to the inverse of the square of the distance $D$ between the slit and the sensor.

D2) the distribution of light intensity on the screen is described, under Fraunhofer’s conditions, by the equation:

$$I(\theta) = I_0 \left( \frac{\text{senn} \theta}{z} \right)^2 \text{ where } z = \frac{\text{senn} \theta}{\lambda}$$

and $a$ is the slit width, $\lambda$ is the light wave length, $\theta$ is the angle that individuates the point on the screen with respect to the centre of the distribution (Figure 2). $\theta$ is related to the transverse position $y$ on the screen by the simple relation $y = D \tan \theta$ ($\theta = 0 \Rightarrow y = 0$ corresponds to the centre of the distribution).

Fig. 1. Distribution of light intensity for a slit of width 0.01 mm, $\lambda = 630-680$ nm.

Fig. 2. The photons laser beam incides the slit A on S1 screen. The photons transmitted impact upon a screen S2, distant D from S1, where we observe the diffraction pattern. We detect the impacts number with a matrix of detectors $R_0, R_1, \ldots, R_{\text{m}}$, each one of which determines a channel of impact located between $y_{i-1}$ e $y_i$. The oval indicates the activation of the i-th detector.

Fig. 3. VQM simulation of the double slit interference (PERGKSU, 2004).

Experiments at low intensity

The same distributions $I=I(y)$ that are obtained at high intensity, may be obtained when we work with light beams with an intensity so low that the diffraction figure could be interpreted as the result of single photon impacts on the screen. The experiment was effectively proposed in an old film of PSSC (King, 1973), but may also be attempted using simulations (Figure 3).

A quantum mechanical interpretation of the phenomenon implies that we may obtain the distribution of intensity $I=I(y)$ of the diffraction as a result of the impact of single photons on the screen. The situation under analysis is schematised in Figure 2. The photons of a laser beam incides on a first screen S1, with a slit of width $a$. The photons transmitted are collected by a screen S2 which shows the figure of diffraction. S2 is subdivided in $2m+1$ channels of impact, each one located between $y_{i-1}$ e $y_i$. The oval indicates the activation of the i-th detector.

The equation:

$$f(y_i) = I(y_i)/I = N(y_i)/N$$

provides the probability $P(y_i)$ of a single photon impact on the i-th channel.

Interpretative hypotheses

The distribution $I=I(y)$ that we observe is the result of the interaction of photons with the slit, in that: if we cover the slit, we observe nothing on the screen; if we remove S1, only a luminous spot may be seen on the screen. This interaction does not depend upon the material of which the screen S1 is made, in such that the distribution $I=I(y)$ depends solely upon the value of $a$, which is easy to recognise in the experiment.
In addition we must exclude any hypothesis of a deterministic construction of the image, as when the photons that impact upon a channel Ri of the screen are, for example, among those that have passed through the upper/lower semi-slit; in obscuring the other half of the slit (Figure 4) we modify the entire figure and not only the part with which we are dealing. We must then conclude that \( P(A1R_i) \neq P(AR_i) \), where \( P(A1R_i) \) e \( P(AR_i) \) are probabilities that a photon will impact on the channel Ri, which respectively passes through the slit A1 or the entire slit A. Similarly, we reach the conclusion that \( P(A2R_i) \neq P(AR_i) \) and, in addition, that \( P(AR_i) \neq P(A1R_i) + P(A2R_i) \), with an analogous significance of the symbols.

This expresses the fact that the distribution produced from a slit of width \( a \) is not the sum of the distributions obtained with each of the two semi-slits of width \( a/2 \).

We may conclude that all the photons that reach S2 utilise (in some way) the entire slit A to propagate from the source to S2.

**The quantum interpretation**

The case of polarisation may help with the carrying out of the analysis. If a beam of polarised photons incides on two aligned birefringent crystals, one direct and the other inverse (Figure 5), the propagation of the photons occurs in a superposition of states. Each one of these corresponds to different potential alternatives where each photon may be detected, interposing a screen for example on one of the paths (ordinary or extraordinary), which correspond with cases of mutually exclusive states of orthogonal polarisation.

If we apply the same criteria to the case of photons transmitted through the slit A, the vector \( W_{AR_i} \), which represents the state \(<\text{passage through A – detection in } R_i>\), must be a linear combination of \( W_{A1R_i} \) and \( W_{A2R_i} \), vectors of state in which the photon may be detected in \( R_i \) if \( A2 \) or \( A1 \) are obscured:

\[
W_{AR_i} = \frac{1}{\sqrt{2}} (W_{A1R_i} + W_{A2R_i})
\]

with the hypothesis that the slit is uniformly illuminated.

To find the probability of detecting the photon in \( R_i \) we evaluate the square module of the scalar product

\[
u_{R_i} W_{AR_i} = \frac{1}{\sqrt{2}} (\Psi_{IR_i} + \Psi_{2R_i}),
\]

with: \( \Psi_{IR_i} = u_{R_i} W_{AR_i} \) and \( u_{R_i} \) vector which represents the detection of the photon in \( R_i \).

The slope of the distribution of light intensity on the screen is given by:

\[
P(AR_i) = |u_{R_i} W_{AR_i}|^2 = 1/2 |\Psi_{IR_i} + \Psi_{2R_i}|^2
\]

This is a non-uniform distribution, which is, however, completely different from expression (1).

In the case in which we consider a sub-division of the slit in \( n \) parts (Figure 6), the written formulas may be easily generalised thus obtaining a better fit of experimental distribution:
\[ W_{AR_i} = \frac{1}{\sqrt{n}} \sum_n W_{AJ|R_i} \quad \text{and} \quad u_{R_i} W_{AR_i} = \frac{1}{\sqrt{n}} \sum_n \Psi_{j|R_i} \]

\[ P(R_i) = |u_{R_i} W_{AR}|^2 = \frac{1}{n} |\sum_n \Psi_{j|R_i}|^2 \]  

(2)

Given that for large \( D \) the experimental distribution depends upon \( 1/D^2 \), it is expected that \( \Psi_{j|R_i} \sim \frac{\phi(r_{ij})}{D} \), with \( \phi(r_{ij}) \) proportional to a momentum autofunction, in fact the momentum \( p \) of photons is sufficiently well defined that it can be assumed: \( \phi(r_{ij}) \sim \exp\left[\frac{i p \cdot r_{ij}}{\hbar}\right] \), with \( r_{ij} \) module of the vector \( AR_i \) and \( \hbar \) is the Planck constant. With these positions in the relation (2), it is obtained:

\[ P(R_i) = \frac{1}{n} |A/D|^2 |\sum_n \exp\left[\frac{i p \cdot r_{ij}}{\hbar}\right]|^2 \]

Passing to the limit for \( n \to \infty \), we have:

\[ P(R_i) = 1/n |A/D|^2 |\sum_n \exp\left[\frac{i p \cdot r_{ij}}{\hbar}\right]|^2 = \frac{1}{2\hbar \sin \theta} \frac{I(\theta)}{I_{\max}} \]

This distribution of probability reproduces the experimental distribution under Fraunhofer’s conditions (1).

**Fig. 6. The construction of the mutually exclusive elementary alternatives.**

**Conclusions**

With a model based upon the principle of superposition, the analysis of mutually exclusive alternatives for photons passed through a slit allows a quantum mechanic interpretation of diffraction. We have built a description of diffraction that identifies all possible states of superposition that, with the same weight, intervene to determine the state of superposition that represents the propagation of photons. We therefore determine the probability of detecting photons in the different zones of a screen reproducing the distribution of intensity that is detected experimentally under Fraunhofer’s conditions. In this sense the result may be considered as his interpretation.

The model is easily extendible to the phenomenon of the interference of a thin surface.

**List of references**


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