Published on
***Science & Education*, 11, No.6, 2002, pp.601-617**

The substantivalist view of spacetime proposed by Minkowski and its educational implications

OLIVIA LEVRINI

Physics Department, University of Bologna - ITALY
E-mail: levrini@df.unibo.it

ABSTRACT

The geometrical interpretation of general relativity provides the formalism with intuitive imagery (Chandler, 1994). Such an interpretation often presupposes a substantival space: a space taken to be a real entity whose existence is independent of the matter contained. Nowadays an image of space-container seems to have wide acceptance among physicists. Special relativity is, however, usually still taught as the theory which overthrew Newton's absolute concepts (among which is the idea of a space-container).

This inconsistency is considered here. Special relativity can also be interpreted substantivally, as Minkowski did in 1908. His substantivalism is the key to laying out an internally coherent substantivalist line running from Newtonian mechanics to general relativity.

Another plausible interpretative line, namely the ‘relationist line’, will be mentioned. It will allow us to conclude with remarks concerning the cultural and educational implications of a consistent interpretative apparatus organised in several interpretative lines.

INTRODUCTION

“*Space acts on matter, telling it how to move. In turn, matter reacts back on space, telling it how to curve*” (Misner *et al.*, 1970) represents a deservedly famous expression of the qualitative content of general relativity (GR) and, in particular, describes the basic ideas of the so-called geometro-dynamical interpretation of the theory. As Chandler has pointed out (Chandler, 1994), the idea of a curved four-dimensional space[1] is counterintuitive only at first glance: there are in fact “ways to re-educate our intuitions” and think about GR as an intuitively comprehensible theory. Chandler proposes that one’s intuitions be adjusted by an awareness that the curvature of space is a manifestation of gravity; in her opinion a geometrical image of gravity has more persuasive power than the “invisible, immaterial ‘force of gravity’ demanded by the Newtonian universe”.

Chandler’s proposal, however acceptable, avoids a delicate issue which has always been at the centre of discussions, and will continue to re-emerge in physics: what is the *real nature* of space? Is it a physical object endowed with substantiality (the ‘space-container’ or ‘substantival space’) or is it no more than a set of formal relations among objects or possible positions of objects (if we refer to space), or among events or possible events (if we refer to spacetime), constructed by human reason to organise or ‘comprehend’—as Einstein 1954a puts it—the factual world (‘relational space’)?

Chandler assumes—as do Misner, Thorne and Wheeler—a substantival space, conceived of as a real entity whose existence is independent of the contained matter, though the two interact.

The geometro-dynamical interpretation of GR has nowadays wide acceptance among physicists and strong influence on teaching, maybe because it cannot be denied that the assumption of a real space introduces a strong criterion for interpreting the basic principles of GR (Levrini, 2002). This point raises a cultural and educational issue: special relativity (SR) is usually taught as the theory which overthrew Newtonian absolute concepts (among which is the idea of a space-container), whereas GR is said, often without justification, to find its true meaning in a space viewed as both absolute and substantival. Is this new absolute space so different from Newtonian space to make such a choice acceptable? Or does the overall presentation of spacetime theories in fact have inherent inconsistencies? Is it possible to reconstruct consistent interpretative lines that go through spacetime physics from classical mechanics to GR?

The article will address the questions by:

- presenting the debate between substantivalism and relationalism in regard with the concept of space in physics and the main arguments supporting the idea that nowadays the image of a 'space-container' is legitimate both from a philosophical and a physical point of view;
- presenting a reconstruction of the original view of SR proposed by Minkowski in his Cologne lecture "Space and Time" (1909) and arguing in what sense Minkowski's interpretation can be considered a substantivalist interpretation of SR and, consequently, to what extent it represents the key to building a consistent substantivalist line running from Newtonian mechanics to GR;
- comparing Minkowski's and Newton's substantivalism and discussing Einstein's criticisms against absolute space.

The present study arises from the consideration that the current debate on the foundations, history and philosophy of spacetime is interesting and rich from a cultural and educational point of view, but it is not exploited enough for improving teaching. Indeed, even though Minkowski's interpretation has been studied by the research in history and philosophy of physics (see, for example, Galison, 1979), no meaningful trace of these results can be found in teaching, as a glance at university textbooks and schoolbooks shows. The teaching tradition, indeed, seems to follow the Einstein's original interpretation which grounds SR on the necessity of defining operationally space and time (Einstein, 1905; Bridgman, 1949) and, hence, of giving them a relational meaning. No other possible interpretation of the theory is usually mentioned in teaching and students are encouraged to believe that relativity has definitively closed the debate on the concept of space.

The reconstruction of Minkowski's alternative interpretation we shall present is meant to be an example of interconnection between research in history-philosophy of physics and in physics education aiming at providing teachers and teacher trainers with historical-philosophical hints for:

- situating critically SR within the current debate on the concept of space and the current interpretations of GR;
- stressing physics as an intellectual product, characterised by the co-existence of several possible interpretations of the same formalism, whose success is strongly influenced by the overall cultural context;
- giving students the opportunity to elaborate criteria for comparing critically different interpretations in order to recognise the one most congenial to their world view and, hence, to assume an active and personal attitude toward their own learning.

THE DEBATE ON THE CONCEPT OF SPACE IN PHYSICS: THE MAIN CRITICISMS AGAINST SUBSTANTIVALISM

The main terms of the debate on the concept of space in physics find their first systematic definition with the dispute between Newton and Leibniz. If the first physical theory explicitly concerning a substantival space has to be ascribed to Newton, the first relationalist argument is the "principle of the identity of indescernibles" elaborated by Leibniz against Newton's absolute space (Alexander, 1956).

> "Space is something absolutely uniform; and, without the things placed in it, one point of space does not absolutely differ in any respect whatsoever from another point of space. Now from hence it follows (supposing space to be something in itself, besides the order of bodies among themselves) that 'tis impossible there should be a reason, why God, preserving the same situations of bodies among themselves, should have placed them in space after one certain particular manner, and not otherwise; why every thing was not placed the quite contrary way, for instance, by changing East into West. But if space is nothing else, but the possibility of placing them, then those two states, the one such as it now is, the other supposed to be the quite contrary way, would not at all differ from one another. Their difference therefore is only to be found in our chimerical supposition of the reality of space in itself. But in truth the one would be the same thing as the other, they being absolutely indiscernible; and consequently there is no room to enquire after a reason of the preference of the one to the other." (Leibniz, in Alexander, 1956)

Leibniz's argument can be considered a relationalist argument inasmuch as absolute space is criticised by comparing its ontological necessity with respect to a relational space: a space meant as "the order of bodies among themselves" or "nothing else, but the possibility of placing them". Space conceived of as a real object is considered by Leibniz a "chimerical supposition" because it introduces a difference between configurations to which no physical (ontological) property can be attached. On the other hand, space meant as nothing but an order among bodies leads indiscernible configurations to be interpreted as identical models of the natural world. Since substantivalism (or realism) is criticised in regard to space being ontologically necessary or proper, the argument is considered pure metaphysical (Sklar, 1974; Nerlich, 1994).

The principle of the identity of indescernibles still represents the best argument against substantivalism. It, indeed, can be translated into the context of GR by replacing Leibniz' spatial displacements with diffeomorphisms (Norton, 1992; Sklar, 1974). Substantivalists are nowadays criticised by arguing that their position leads them to deny the so-called "Leibniz equivalence": diffeomorphic models represent the same physical world. But by denying Leibniz equivalence, substantivalists must also accept a radical form of indeterminism, because a corollary of Leibniz equivalence states that diffeomorphic models can differ in properties that remain undetermined (the "hole argument"[2]). Therefore substantivalists must not only accept that diffeomorphic models represent different worlds, but also that some of these worlds have undetermined properties.

Although the "hole argument" is considered the strongest criticism against substantivalist, it does not represent a definitive argument and the debate is still open[3]. In particular "holes in the hole argument" are pointed out by Nerlich thanks to a "metaphysical" analysis of the argument fairly close to the historical Leibniz, and to an "extensional" analysis aimed at seeing "the issue as a cost-benefit calculation in theory choice, pricing, as it were, one set of metaphysically innocent theoretical entities against another". Nerlich's conclusion is that "whichever way we interpret the argument, we find no strong reason to abandon realism" (Nerlich, 1994; p.206).

Among physicists the substantival space is usually considered old-fashioned: a reformulation of Newtonian absolute space. They criticise it, as Mach and Einstein did, on grounds of its unobservability and, hence, consider it a "conceptual monstrosity" (Mach, 1912). To this objection, a 'substantivalist' usually answers that, since GR expresses a physical-causal interaction between matter and space, the latter, even if not directly observable, is not causally inert. Its existence manifests itself in the fact that, if postulated, it can convincingly explain observable gravitational effects. This kind of argument is known as "inference to the best explanation" (Boniolo & Dorato, 1997): existence is ascribed to theoretical entities, even if these are not directly observable, because their postulation allows the formulation of the explanation providing the most persuasive interpretation of the empirical data (Friedman, 1983; Nerlich, 1994).

The best physical argument against substantivalist arises from the physicists engaged in the research of the great unification theory. For them, as Weinberg points out, the substantival identification between spacetime and differentiable manifold is an obstacle in the unification: "too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics"[4] (Weinberg, 1972). The current state of the research does not provide, however, a definitive physical result against substantivalism, because at moment the unity of physics can be considered as much metaphysical as the cognitive need of a substantival space for interpreting GR geometro-dynamically.

In conclusion we can say that a substantival (or real) space has not been definitely removed from physics, even if it has been criticised by outstanding philosophers and physicists. As a consequence, it can be legitimately assumed at the basis of one possible interpretation of GR, alternative to other legitimate interpretations based on a relationalist conception of space (Levrini, 2002). Nevertheless, the teaching of GR according to the geometro-dynamical interpretation demands teachers to re-think carefully about classical mechanics and SR in order to prevent students from dealing unconsciously with a new spatial container, after that they have been convinced that Einstein, thanks to relativity, allowed physics to overcome absolute space. The problem can be avoid if SR is discussed also from a Minkowskian perspective: the perspective on which geometro-dynamical interpretation is naturally grounded.

MINKOWSKI'S LECTURE "SPACE AND TIME"

Many university textbooks[5] and the few Italian schoolbooks that address Minkowski's spacetime give the topic only marginal presentation, in which spacetime has the role of a simple and useful representative tool. In fact it is used to show how a visible, intuitive form can be given to concepts like the relativity of simultaneity, length contraction and time dilation, for instance, already expressed algebraically in the Lorentz transformations.

This was not, however, the meaning Minkowski originally attached to spacetime geometry in the talk "Space and Time" he gave in September 1908 (published in 1909). Minkowski's meaning is expressed in his introductory comments:

> "The views of space and time which I wish to lay before you have sprung from the soil of experimental physics, and therein lies their strength. They are radical. Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality"[6] (Minkowski, 1909).

His general aim was to show "how it might be possible, starting from the accepted mechanics of the present day, along a purely mathematical line of thought, to arrive at changed ideas of space and time"[7].

The central problem addressed in the article is the dishomogeneity between two groups of transformations—spatial rotations and Galilean transformations—which preserve the form of the differential equations of classical mechanics. Minkowski sees significant differences between these groups:

* the first group expresses *spatial* homogeneity and isotropy, properties to which *geometrical* meaning is attached;
* the second group expresses classical *temporal* characteristics (the existence of an absolute time and the freedom of choosing the direction of the 'upper half' of the world) and the *physical* assumption that no phenomenon allows reference frames to be told apart.

The uneasiness provoked by these differences induce him to interpret the four-dimensional geometry introduced by Poincaré (Poincaré, 1905; Poincaré, 1906) as the road to solve the inhomogeneities present in classical physics. Minkowski's specific aim is to re-interpret the Lorentz group as the more general group of transformations, of which the two classical ones represent special cases. As a result the formal structure acquires that unitary and organic form which makes it possible to understand "what the requirement of orthogonality in space" (i. e. the requirement that physical laws be invariant for homogeneous linear transformations preserving the form of the expression $x^2+y^2+z^2$) expressed by the first group has to do "with the perfect freedom of the time axis in an upward direction" implied by the second[8].

To establish the connection between the two groups, Minkowski considers, in the x,t plane, the invariant equation $c^2t^2 - x^2 = 1$, where *c* is a parameter to which no special meaning is attached at first. A new co-ordinate system x', t', in which the expression retains the same form, is then constructed geometrically (see fig. 1).

Through apparently artificial steps he arrives at the conclusion that the requirement that hyperbola equation be invariant singles out, for every value of *c*, a well defined group of linear transformations. The group obtained is denoted G_c to indicate dependence on *c*. The complete classical group of transformations—the *Galileo group*—G_∞ in Minkowski's notation, is re-obtained (Minkowski, 1909) if *c* is made infinite.

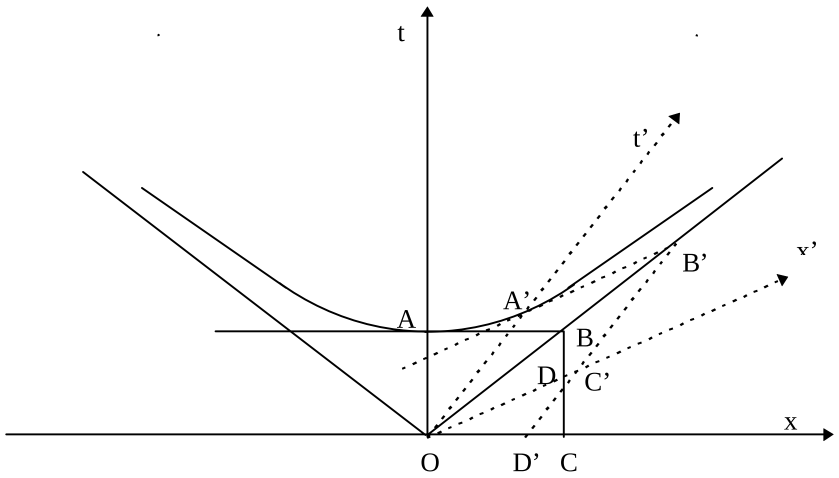

Fig. 1. The diagram illustrates Minkowski's construction of new axes t' and x' from the hyperbola now known as the 'calibration curve': t' is constructed in analogy to t as the axis intersecting the hyperbola; x', in analogy to x, as the line parallel to the line tangent to the hyperbola in the point of intersection between the temporal axis and the hyperbola itself (Minkowski, 1909).

The four-dimensional geometry, invariant under G_c, expresses various different requirements, which together form the basis of Minkowski's substantival view. It satisfies the aforementioned requirement of a formal structure being able to absorb the particular differences. It also expresses a harmony between physics and geometry and the possibility of anchoring physical theories to the idea that world must be absolute, in other words independent from observers' views. Let us consider these points in detail.

The geometrization of physics

Minkowski emphasises that the new four-dimensional world is not a product of pure formal speculations, but arises from the natural world, or rather has "sprung from the soil of experimental physics". So he urges one to "visualize the state of things by the graphic method"[9]. Minkowski points out that a visualisation naturally happens in space and time rather than just in space, as "the objects of our perception invariably include places and times in combination. Nobody has ever noticed a place except at a time, or a time except at a place"[10].

To make this idea explicit and remind the reader that this way to perceive the factual world is the natural one, Minkowski uses the term *world-point* to denote "a point of space at a point of time"[11]. The *world* is the set of all the possible values of x, y, z , t, and the world-points representing "the career"[12] of an object constitute the *world-line*.

Minkowski's project to geometrize physics hence acquires natural legitimacy, inasmuch as its roots lie in the phenomenal world, of which it attempts an explanation. Owing to its properties of symmetry, generality and invariance (Galison, 1979), four-dimensional geometry becomes the privileged way to free knowledge about the factual world from the imperfections and hesitations arising from our instinctive attachment to the direct and approximate way of perceiving and observing nature: "physical laws might find their most *perfect* expression as reciprocal relations between these world-lines"[13].

By looking at the phenomenal world from a structured four-dimensional perspective, therefore, can physical laws be expressed simply. Indeed in three-dimensional classical space they leave what is no more than a complicated projection, "a shadow". Space by itself and time by itself are, for Minkowski, only shadows (in three dimensions or just one) of the real world, "shadows" allowed "to fade away" by mathematical thought.

The invariant as an essence of the world

If this geometrized world or 'spacetime' can be derived "from the totality of natural phenomena"[14], it is also true that when this is done, it is "by no means uniquely determined by the phenomena"[15]. The essence of the theory is the invariance of the geometrical structure under the group G_C; the name best suited to the principal meaning of the theory is therefore "*the postulate of the absolute world* (or briefly, the world-postulate)"[16] rather than the "*relativity-postulate*"[17].

This geometrical structure is hence preserved, independently of the phenomena and observers. It is the structure of the entire world that includes every observer, everywhere and at all times.

This point does not mean we can no longer speak about relativity, or about systems associated with observers viewing phenomena from their particular perspective; but we can only do so insofar as we give up the four-dimensional world and refer to it through its three-dimensional projection in space and its one-dimensional projection in time.

MINKOWSKI'S SUBSTANTIVALIST VIEW

As has been said, spacetime is neither intended by Minkowski to be a mere graphical representation of Einstein's SR, nor a pure mathematical abstraction. It expresses a global and radical view of the complex relation between physics, geometry and the natural world, deriving from the belief in an existing "pre-established harmony between pure mathematics and physics"[18].

Minkowski's spacetime is widely viewed as a geometrical translation of the interdependence between space and time, already given algebraic expression in the Lorentz transformations. This interdependence would have suggested the idea of representing events in a four-dimensional space and hence of manifesting the impossibility of keeping space and time apart.

Minkowski's view of spacetime, however, goes beyond the mere re-unification of space and time in an invariant expression. It can be arrived at putting together the three key concepts expressed in Minkowski's introductory comments: spacetime *derives from the phenomenal world*, it is *real* and *observer-independent.*

Minkowski looks at the four-dimensional geometry – the 'world'- as the primitive entity and the properties of material processes and events (physics) have to be explained and predicted by relating them to the geometrical structure within which they are 'contained'.

The behaviour of "substantial points"[19] (points occupied by physical objects) must therefore be studied in terms of invariant relations between their world-lines, for it is in such relations that the laws of physics "reveal their inner being in full simplicity"[20]. When Minkowski refused to think about his 'world' as a primeval abyss ("yawning void"[21]), he was saying that his world has no mythical overtones, but that it both explains and was modelled on the phenomena. It manifests the true essence of reality because, in its four-dimensional dress, it avoids both the mere relativism of space and of time and reduces three-dimensional space to the role the "shadows" have in Plato's cave. "Now you know why I said at the outset that space and time are to fade away into shadows, and only a world in itself will subsist"[22].

Minkowski establishes a *literal* identification of the natural structure of the real world with the mathematical structure of spacetime. Minkowski's position is, in other words, based on the substantivalist assumption that the real world *is* a four-dimensional manifold, so that the latter becomes a *real* object. Such a structure – literally identified with the natural world – must be invariant, observer-independent and hence absolute.

To complete Minkowski's view it must be added that, being geometrical and thus rational, the intrinsic structure of the world assumes forms that human beings can recognise and comprehend. This means that scientists have powerful aesthetic criteria as symmetry, invariance or generality at their disposal for evaluating the degree to which theory 'fits' the natural world. These indeed are the forms through which the harmony—which transcends knowledge (being *pre*-established)—between pure mathematics and nature[23] manifests itself in "the soil of experimental physics".

MEANINGS OF THE EXPRESSION 'ABSOLUTE SPACE': NEWTON AND MINKOWSKI COMPARED

Minkowski, then, proposes a new "absolute world", only three years after Einstein's "Zur Elektrodynamik bewegter Korper" (1905), which seemed to have removed from physics all that could not be given an operational definition. The new absolute nevertheless does not represent a step backward with respect to Einstein, inasmuch as Minkowski's spacetime embodied the physical conquests of SR and his geometrization of physics paved the way for GR. To understand how such an absolute spacetime differs from Newton's space, let us analyse the different meanings that can be associated to the expression 'absolute space' when referred to Newtonian mechanics. The analysis will allow us to distinguish what can be considered a real conquest of scientific thought due to SR from what represents an expression of a particular world view and, as such, must be evaluated appealing to criteria from outside Physics.

When referred to Newtonian mechanics, the expression "absolute space" can assume at least three different meanings:

1. substantival, or existing in itself, container;
2. privileged frame of reference;
3. invariant, or independent from the frame of reference, distance among bodies.

The principle of relativity extended to all physical phenomena and the relativistic effect of length contraction are the physical arguments provided by Einstein in his paper about SR against respectively the second and third meanings of the Newtonian absolute space (Einstein, 1905).

With respect to such meanings, Minkowski's spacetime represents an irreversible evolution of Newton's absolute space due to SR. In particular, as far as the third meaning is concerned, the four-dimensional world revealed, in Minkowski's perspective, by SR implies the invariance to be related not to the spatial dimensions – taken separately from the temporal one – but to the spacetime interval between two different events. In this sense Einstein's criticism based on the relativistic effect of length contraction is considered by Minkowski as a Newtonian residual of taking space and time apart.

As far as the second meaning is concerned, Minkowski doesn't agree with Newton about the necessity to attach a privilege to only one frame of reference and, in accordance with the relativistic view, considers all the inertial frames perfectly equivalent, even though they still maintain a privilege with respect to the non-inertial ones. The difference between Newton and Minkowski in regard to the second meaning is, however, not only related to the "number" of the privileged frames, but also to the nature of the privilege. For Newton his absolute space is privileged as 'actual', 'non-apparent', 'non-relative' to the observation. In this sense the privilege is the direct consequence of its being the only space endowed with 'reality'. The same kind of privilege, i.e. of 'absoluteness' is ascribed by Newton to quantities as position, velocity, time, when referred to the absolute space. For Minkowski the privilege does not arise directly from the substantival character of spacetime, but from the structural geometrical properties of spacetime: the property of having non-trivial inherent symmetries thanks to which the physical world is seen by every inertial observatory as identical.

As far as the first meaning of absolute space is concerned, SR itself cannot be considered as a theory providing new arguments with respect to the arguments used in the traditional debate about substantivalism. As a "proof" that the debate substantivalism-relationalism belongs to a meta-physical level, the refusal of every formulation of substantival space (Newton's or Minkowski's one) is argued by Einstein[24] prevalently in epistemological papers:

> "space as opposed to 'what fills space' has no separate existence [...]. There is no such thing as an empty space, *i.e.*, a space without field. Spacetime does not claim existence on its own, but only a structural quality of the field" (Einstein, 1952, 375).

The relationalist position is in Einstein intrinsically related to an image of physics as a "free play of images and concepts": thought is free to create concepts and systems of concepts and their only reason to exist lies in their function of "representing the complex of our experiences; otherwise they have no legitimacy"[25] (Einstein, 1922). From this perspective not all theoretical terms have to be taken literally by endowing them with substantiality. Even when Einstein stated to be a rationalist and expressed respect for mathematics, his attitude in speaking about SR was strongly conditioned by the need of taking observational terms apart from theoretical ones (Schilpp, 1949; Friedman, 1983), whose "earthly lineage" is granted if and only if a 'correspondence rule', an 'operational definition' exist.

> "Why is it necessary to drag down from the Olympian fields of Plato the fundamental ideas of thought in natural science, and to attempt to reveal their earthly lineage? Answer: In order to free these ideas from the taboo attached to them, and thus to achieve greater freedom in the formation of ideas or concepts. It is the immortal credit of D. Hume and E. Mach that they, above all other, introduced this critical conception." (Einstein, 1952; 365).

The difference between such an epistemological position and Minkowski's can be marked by underlining, for instance, that for Minkowski the harmony between mathematics and physics is an assumption, while for Einstein it is a post-judice: "The eternally incomprehensible thing about the world is that it can be understood". By comprehensibility he means

> "an order among sense impressions, this order being produced by the creation of general concepts, relations between the concepts and by definite relations of same kind between the concepts and sense experience" (Einstein, 1954, 292).

Einstein's wonderment should be compared with the emphasis Minkowski gives to the invariance of world structure and natural phenomena under the group G_c, which he offers as an example of harmony between mathematics and nature.

Even though the reserves expressed by Einstein with respect to Minkowski spacetime immediately after its presentation are widely documented (Pais, 1982), it is also a matter of fact that during the construction of GR, Einstein started to realise the relevance of a four-dimensional view and of the geometrical approach with respect to the original algebraic one. So that, after months of "hard work" on tensorial formalism he finally said:

> "I have become imbued with great respect for mathematics, the subtler part of which I had in my simple-mindedness regarded as pure luxury until now." (Einstein, in Pais 1982)[26].

These words can be interpreted as the first signals of what is considered, for example by Holton, a real turning in Einstein's thought from the juvenile empiricism toward a more mature realism (Holton, 1973). Surely they indicate the fact that Einstein, working on GR, arrived at reconsidering the role of mathematics as a powerful and unifying tool for constructing physics knowledge.

The influence of Minkowski on Einstein - the world we perceive is only a shadow of the real four-dimensional world - is evident in a letter to Michele Besso (July, 1952). Here Einstein reproaches the friend by saying:

> "You do not take seriously the four dimensions of relativity, but consider however the present as it were the only reality. What you call 'world' corresponds, in physical terminology, to 'space sections', to which the theory of relativity – even special relativity – denies objective reality" (Einstein, in Holton, 1973).

Those "space sections" are the projections of Minkowski's talk, the shadows that mathematical thought has revealed to be partial perceptions of reality.

After publication of GR, Einstein often recognised his debt to Minkowski, but the two positions didn't arrive at superposing each other in a firm and decisive way. Einstein never lost a certain diffidence toward a pure substantival conception of space. The 'rationalist' Einstein ended up by saying that physics was the attempt of conceptual construction of a *model* of the *real world* and of its law-governed structure. What changed in Einstein's thought, also thanks to Minkowski paper, was the idea of model, which assumed also the features of a *mathematical* model able to codify knowledge about natural world and its structure.

In conclusion we can say that, although physicists have tried to eradicate the absolute space from physics, some meanings of the expression are still not only legitimate but also seem to arise from deep cultural and cognitive needs. In particular the substantival character of space cannot be removed radically, nor corroborated with purely physical arguments: for the time being it must be criticised or supported using argumentative tools taken from other fields such as philosophy or history of physics.

EDUCATIONAL IMPLICATIONS

Physics can be seen as a body of knowledge organised within a sound logical-mathematical structure which has, since Galileo, been systematically studied, developed and expanded as a container in which to accumulate knowledge about the physical world. Though the importance of formal structure can hardly be doubted, it is inconceivable that physical knowledge can be completely enclosed in it: the stock of knowledge acquired or constructed to give meaning to the formalism represents an *interpretative apparatus* which can be only partly formalised, because the complex relation between mathematics, observation, experiments, thought and reality from which the various interpretations derive can only be partly formalised.

The interpretative apparatus is made up of aspects of reality and experimental results studied together with the theoretical formal structure on the basis of criteria concerning the relation between mathematics and nature. Then there are ideas, concepts and meta-physical aspects (religious, cultural, aesthetic), which also belong to the interpretative apparatus of the theory, that cannot be formalised, even if they are important in giving meaning to the theory. Unlike the analysis of the formal structure, the analysis of the interpretative devices aiming at making it internally organised and self-consistent has been not always translated into coherent teaching paths. In a word, history and philosophy of physics have been not exploited enough for improving teaching.

The teaching of spacetime physics is representative of this situation: the different meanings assumed by the relativistic theories during the evolution of physical thought are nowadays taught according to the temporal order in which they were proposed and accepted in the scientific community, without taking into account the results obtained by the current researches in history and philosophy of Relativity. So that, even though nowadays the geometro-dynamical interpretation of GR has wide acceptance among physicists, SR is usually still taught as the theory which overthrew Newton's absolute concepts, among which the idea of a substantival space.

The present study is meant to be a contribution to making the historical-philosophical debate on spacetime a support for teaching. In particular it should encourage teachers and teacher educators:

- to situate SR content knowledge within a scientific debate still open since several alternative interpretations of the theory are all equally possible and legitimate;
- to present the acceptance of and the preference for one interpretation instead of another among physicists as an historical ‘contingence’ and not as a definitive, logical necessity;
- to remove the idea of physics as an unquestionable body of knowledge by showing in what sense it can be always revisited in the light of more recent physics knowledge and reread from different perspectives evolving in time.

In other worlds, we see the study as a contribution for teachers and teacher educators coherent with the idea that “teaching controversy” (Bevilacqua, 1999) can in fact help students to focus on the peculiar aspects of each interpretation and to elaborate logical, cultural, rhetorical, cognitive instruments for comparing different perspectives and for expressing their own preferences.

Furthermore, as several researches have pointed out (see, for example, Matthews, 1992; Matthews, 1994), a teaching approach based on history and philosophy of physics can have important implications for education since:

- it fosters a deeper understanding of content knowledge, inasmuch as it allows formulas to gain a cognitive and cultural meaning;
- it can capture the interest of a larger number of students, since it makes physics more human;
- it fosters a critical attitude toward learning and can make classes more active.

The concept of space, used in the present study as criterion for analysing the controversy between the different interpretations of SR, has been used as leading thread in the design of four different conceptual paths for teaching spacetime physics at secondary school level (Levrini, 1999; Levrini 2000). At present we are empirically investigating the educational implications of the paths both with prospective physics teachers and with 18-19 year old secondary school students. As a first qualitative outcome we can say that the possibility to anchor the discussion about the interpretations of SR to a primitive question ('What is space?') is very effective in engaging prospective teachers and students in their own learning process: they recognise the question as a real cultural problem and, in the debate on the concept of space, they find the tools for justifying their double-faced attitude of diffidence and curiosity toward a theory that they wish to recognise both as a revolution and as the natural development of classical physics.

ACKNOWLEDGEMENT

The present study is part of my PhD dissertation thesis. So I wish to thank first of all my supervisors, Silvio Bergia and Nella Grimellini Tomasini, for their support and their always stimulating comments and suggestions on the study. I also want to express my gratefulness to Carla Casadio, Michela Clementi and Enrico Giannetto for the fruitful and helpful discussions. Their ideas and opinions have been really precious. And thanks to Alexander Afriat and Barbara Pecori for their sharp and exact remarks, as well as for their competence and patience in improving editing for the English translation.

NOTES

1. Here *space* is taken to mean *spacetime*—as in Misner, Thorne and Wheeler—to emphasise the continuity between different spacetime theories.
2 For more details about the "hole argument" see, for example, Earman & Norton, 1987; Norton, 1992.
3 See, for example, Friedman, 1983; Nerlich, 1994.
4. It is worth specifying that Weinberg's critics focus above all on a presumed confusion between physics and history of physics. Indeed, the complete quotation is: "As long as it could be hoped, as Einstein did hope, that matter would eventually be understood in geometrical terms, it made sense to give Riemannian geometry a primary role in describing the theory of gravitation. But now the passage of time has taught us not to expect that the strong, weak, and electromagnetic interactions can be understood in geometrical terms, and too great an emphasis on geometry can only obscure the deep connections between gravitation and the rest of physics. [...] the great thing is not to confuse physics with history, or history with physics."(Weinberg, 1972).
5. See, for example, Rindler (1977), Resnick (1968), Schutz (1985).
6. "Die Anschauungen über Raum und Zeit, die ich Ihnen entwickeln möchte, sind auf experimentell - physikalischen Boden erwachsen. Darin liegt ihre Stärke. Ihre Tendenz ist eine radikale. Von Stund an sollen Raum für sich und Zeit für sich völlig zu Schatten herabsinken und nur noch eine Art Union der beiden soll Selbständigkeit bewahren."
7. "[...] wie man von der gegenwärtig angenommenen Mechanik wohl durch eine rein matematische Überlegung zu veränderten Ideen über Raum und Zeit kommen könnte."
8. "Was hat nun die Forderung der Ortogonalität im Raume mit dieser völligen Freiheit der Zaitachse nach oben hin zu tun?"
9. "[...] die Verhältnisse graphisch zu veranschaulichen suchen."

10. “Gegenstand unserer Wahrnehmung sind immer nur Orte und Zeiten verbunden. Es hat niemand einen Ort anders bemerkt als zu einer Zeit, eine Zeit anders als an einem Orte.”
11. “[...] einen Raumpunkt zu einem Zeitpunkt”
12. “Lebenslauf”.
13. “[...] die physikalischen Gesetze ihren *vollkommensten* Austruck als Wechselbeziehungen unter diesen Weltlinien finden dürften.” - Italics mine.
14. “[...] aus der Gesamtheit der Naturerscheinungen [...].”
15. “[...] durch die Escheinungen keinesweg eindeutig festgelegt.”
16. “*Postulat der absoluten Welt* (oder kurz Weltpostulat).”
17. “*Relativitätspostulat*”.
18. “[...] eine prästabilierte Harmonie zwischen der reinen Mathematik und der Physik”.
19. “substantiellen Punkt”.
20. “[...] die [...] Verhältnisse ihr inneres Wesen voller Einfachheit erst in vier Dimensionen enthüllen”.
21. “Um nirgends eine gähnende Leere zu lassen”
22. “Sie erkennen, weshalb ich am Eingange sagte, Raum und Zeit sollen zu Schatten herabsinken und es soll nur eine Welt an sich bestehen.”
23. Minkowski talks about a “pre-established harmony between pure mathematics and nature” in Draft RZ 4, p.22 labelled 20 (see Galison, 1979)
24. Einstein’s conception of space, as well as his epistemological view, presents several interpretative problems (see, for example, Kostro, 1992). In this context we ascribe to Einstein a relationalist conception of space because Einstein himself has most firmly stated to prefer this one (Einstein, 1952; 1954).
25. The epistemological oscillations in Einstein’s thinking are well know and recognised by himself (Holton, 1996; Schilpp, 1949). Here we refer mainly to Einstein’s epistemological remarks concerning the cultural meaning of relativistic theories.
26. Einstein A., Letter to A. Sommerfeld, 1912, in Pais, 1982.

REFERENCES

Alexander H. G. (Ed.): 1956, *The Leibniz-Clark Correspondence*, Manchester University Press, Manchester.

Bergia, S.: 1990, Strutture e dimensionalità dello spaziotempo: realtà, modello o occasione di formalismo?, in Selleri F., Tonini V. (Eds.), *Dove va la scienza. La questione del realismo,* Dedalo, Bari.

Bergia, S.: 1995, Formulari, interpretazioni, ontologie: il caso delle teorie relativistiche, in Giuliani G. (Ed.), *Ancora sul realismo*, La Goliardica Pavese, Pavia.

Bevilacqua, F.: 1999, Teaching Controversy in Teaching Science, plenary lecture at the Second Joint Conference - *Science as Culture* - Como-Pavia (www.cilea.it/volta99).

Boniolo, G. (Ed.): 1997, *Filosofia della fisica*, Mondadori, Milano.

Boniolo, G. & Dorato, M.: 1997, Dalla relatività galileiana alla relatività generale, in Boniolo, *Filosofia della fisica*, Mondadori, Milano.

Bridgman P. W. (1949), Einstein's Theories and the Operational Point of View, in Schilpp P. A. (Ed.), *Albert Einstein: philosopher-scientist*, The Library of Living Philosophers.

Chandler, M.: 1994, Philosophy of Gravity: Intuitions of Four-Dimensional Curved Spacetime, *Science & Education* **3**: 155-176.

Earman, J. & Norton, J.: 1987, What Price Spacetime Substantivalism? The Hole Story, *Brit. J. Phil. Sci*., **38**, 515-525.

Einstein, A.: 1905, Zur Elektrodynamik bewegter Korper, *Annalen der Physik*, XVII, pp. 891-921 (On the electrodynamics of moving bodies, in Lorentz, H. A., Einstein, A., Minkowski, H., Weyl, H.: 1952, *The principle of relativity. A collection of original memoirs on the special and general theory of relativity*. (with notes by A. Sommerfeld) Dover Publications, New York, pp.37-65).

Einstein, A.: 1916, *Über die spezielle und allgemeine Relativitätstheorie (gemeinveständlich)* (*Relativity, The Special and the General Theory: A Popular Exposition*, 15th ed. R. W. Lawson, trans. London, Methuen, 1954).

Einstein, A.: 1922, *Vier Vorlesungen über Relativitätstheorie*, Vieweg & Sohn, Braunschweig (*The Meaning of Relativity*, Princeton University Press).

Einstein A.: 1952, *Relativity and the Problem of Space*, in Einstein 1954, *Ideas and opinions*, ed. Crown Publisher, trans. Sonja Bergmann, New York.

Einstein, A.: 1954, *Ideas and opinions*, ed. Crown Publisher, trans. Sonja Bergmann, New York.

Friedman, M.: 1983, *Foundations of Space-Time Theories*, Princeton University Press.

Galison, P. L.: 1979, Minkowski's Space-Time: From Visual Thinking to the Absolute World, *Historical Studies in the Physical Sciences*, 10, 85-121.

Giannetto, E.: 1995, Note storico-critiche sul mutamento e il 'realismo': Henri Poincaré, la Relatività Speciale e le Teorie Fisiche, in Giuliani G. (Ed.), *Ancora sul realismo*, La Goliardica Pavese, Pavia.

Holton, G.: 1973, *Thematic origins of Scientific Thought: Kepler to Einstein*, Harvard University press, Cambridge (Mass.).

Holton, G.: 1992, How to Think about the "Anti-science" Phenomenon, in *Public Understanding of science*, 1, 103-128.

Holton, G.: 1996, *Einstein, history, and others passions. The rebellion against science at the end of the twentieth century*, Addison Wesley Publishing Co.

Jammer, M.: 1954, *The History of Theories of Space in Physics*, Harvard University Press, Cambridge (Mass.)

Kattmann, U., Duit, R., Gropengieβer, H., Komorek M.: 1996, Educational Reconstruction -bringing together issues of scientific clarification and students' conceptions-, paper presented to the Annual Meeting of the National Association of Research in Science Teaching (NARST), St. Louis, United States.

Kostro, L.: 1992, An Outline of the History of Einstein's Relativistic Ether Concept, in Eisenstaedt J., Kox A. J. (Eds.), *Studies in the History of General Relativity*, Einstein Studies, Vol.3, Birkhäuser, Boston, United States.

Levrini, O.: 1999, Teaching modern Physics on the basis of a content knowledge reconstruction, Proceedings of the Second International Conference of E.S.E.R.A., Kiel (Germany), 31 August - 4 September, 247-249.

Levrini, O.: 2000, *Analysing the possible Interpretations of the Formalism of General Relativity. Implications for Teaching*, Ph.D. Dissertation, Physics Department, University of Bologna, Italy.

Levrini, O.: 2002, Reconstructing the basic-concepts of General Relativity from an educational and cultural point of view, accepted for publication on *Science & Education*.

Leibniz, G. W. & Clarke, S.: 1717, *A Collection of Papers which passed between the late learned Mr. Leibniz and Dr. Clarke*, London, 1717.

Lorentz, H. A., Einstein, A., Minkowski, H., Weyl, H.: 1952, *The principle of relativity. A collection of original memoirs on the special and general theory of relativity.* (with notes by A. Sommerfeld) Dover Publications, New York.

Mach, E.: 1883, *Die Mechanik in ihrer Entwickelung, historisch-kritisch dargestellt* (*The science of Mechanics: A Critical and Historical Account of Its Development*, La Salle; Open Court, 1960).

Mach, E.: 1912, Preface of 7th ed. *Die Mechanik in ihrer Entwickelung, historisch-kritisch dargestellt*, 1883.

Matthews, M. R.: 1992, History, Philosophy and Science Teaching: the Present Rapprochement, *Science & Education*, 1, p. 11-47.

Matthews, M. R.: 1994, *Science teaching: the role of history and philosophy of science*, New York – London, Routledge, 1994.

Minkowski, H.: 1908, Die Grundgleichungen für die elektromagnetische Vorgänge in bewegten Körpen, in *Abhandlungen von Minkowski*, Hilbert, pp.352-404.

Minkowski, H.: 1909, Raum und Zeit, *Physikalische Zeitschrift*, 10, No.3, 104-111 (Space and Time, in Lorentz, H. A., Einstein, A., Minkowski, H., Weyl, H.: 1952, *The principle of relativity. A collection of original memoirs on the special and general theory of relativity.* (with notes by A. Sommerfeld) Dover Publications, New York, pp.73-96).

Misner, C. W., Thorne, K.S., Wheeler, J.A.: 1970, *Gravitation*, W.A. Freeman and C., S. Francisco.

Nerlich, G.: 1994, *What Spacetime Explains. Metaphysical Essays on Space and Time*, Cambridge University press, Cambridge.

Newton, I.: 1686, Philosophiae naturalis principia mathematica.

Norton, J.: 1992, The Physical Content of General Covariance, in Eisenstaedt J., Kox A. J. (Eds.), *Studies in the History of General Relativity*, Einstein Studies, Vol.3, Birkhäuser, Boston, United States.

Pais, A.: 1982, *'Subtle is the Lord...' The Science and the Life of Albert Einstein*, Oxford University Press.

Poincaré, J. H: 1905, Sur la dynamique de l'électron, *Comptes Rendus de L'Académie des Science.*

Poincaré, J. H.: 1906, Sur la dynamique de l'électron, *Rendiconti del Circolo Matematico di Palermo.*

Poincaré, J. H.: 1913, *Dernières pensées*, Flammarion, Paris.

Resnick, R.: 1968, *Introduction to Special Relativity*, John Wiley & Sons, Inc. -New York- London.

Rindler, W.: 1977, *Essential relativity*, Springer Verlag, New York.

Schilpp, P. A. (edited by): 1949, *Albert Einstein: philosopher-scientist*, The Library of Living Philosophers.

Schutz, R.F.: 1985, *A first course in general relativity*, Cambridge University Press, Cambridge.

Sklar L.: 1974, *Space, Time, and Spacetime*, University of California Press, Berkeley and Los Angeles, California.

Weinberg, S.: 1972, *Gravitation and Cosmology*, J. Wiley and Sons, New York.