THE PROBLEM WITH SO-CALLED FICTITIOUS FORCES

Härtel Hermann; University Kiel, Germany

Abstract
Based on examples from modern textbooks the question will be raised if the traditional approach to teach the basics of Newton’s mechanics is still adequate, especially when inertial forces like centrifugal forces are described as fictitious, applicable only in non-inertial frames of reference.

In the light of some recent research, based on Mach’s principles and early work of Weber, it is argued that the so-called fictitious forces could be described as real interactive forces and therefore should play quite a different role in the frame of Newton’s mechanics.

By means of some computer supported learning material it will be shown how a fruitful discussion of these ideas could be supported.

1. Introduction
When treating classical mechanics the interaction forces are usually clearly separated from the so-called inertial forces. For interaction forces Newton’s basic laws are valid, while for inertial forces especially the 3rd Newtonian principle cannot be applied. There seems to be no interaction force which can be related to the force of inertia and therefore this force is called fictitious and is assigned to a fictitious world.

The following citations from a widespread German textbook (Bergmann Schaefer 1998) may serve as a typical example, which is found in similar form in many other German as well as American textbooks.

- Zu den in der Natur beobachtbaren Kräften gehören schließlich auch die sogenannten Scheinkräfte oder Trägheitskräfte, die nur in Bezugsystemen wirken, die gegenüber dem Fundamentalsystem der fernen Galaxien beschleunigt sind.
  Dazu gehören die Zentrifugalkraft und die Corioliskraft. Durch geeignete Wahl des Bezugsystems kann man diese Kräfte "wegtransformieren", d.h. zum Verschwinden bringen. Daher der Name "Scheinkräfte". (p.99)
  (To the observable forces in nature belong also all the so-called fictitious forces or inertial forces which act only in frames of reference, which are accelerated in respect to the fundamental system of distant galaxies. These include the centrifugal force and Coriolis force. By suitable choice of frames of reference these forces can be transformed to disappear. Hence the name "fictitious forces").

- Um Newtons erstes und zweites Gesetz auch in beschleunigten Bezugsystemen verwenden zu können, hat man die Scheinkräfte bzw. Trägheitskräfte eingeführt. Die beiden Gesetze müssen dann entsprechend erweitert werden, Newtons drittes Gesetz (F_{ab} = - F_{ba}) kann man nach unserem heutigen Kenntnisstand aber nicht in dieser Weise verallgemeinern, denn eine Gegenkraft zu den Scheinkräften wurde bisher nicht entdeckt. (p.241)
  (To use Newton's first and second law also in accelerated reference systems, one has introduced the fictitious forces or forces of inertia. The two laws must then be expanded accordingly. Newton's third law (F_{ab} = - F_{ba}), however, can not be generalized in a similar way, because according to our present state of knowledge a counterforce to the fictitious forces has not been discovered yet.)

The use of fictitious forces in accelerated systems of reference is interpreted as some kind of mathematical trick to be able to use the same Newtonian laws also in such accelerated systems. From experience in classrooms, however, we know that the learners have great difficulty to accept the fictitious character of such forces.

The following two examples (both always seen from an inertial frame of reference) may demonstrate this difficulty. In figure 1 it is assumed that the body with mass m is fixed to left to the wall by a rope and that it is under the action of a force source, pulling to the right. In a state of equilibrium there is no doubt that we have to declare the existence of 2 real forces F of equal seize...
and opposite orientation applied to the body. If the rope to the left is cut and the action of the force to the right will not change during the process of acceleration, nothing will change on the right side of the body. Why do we declare now that the left side has changed and that there is not a real force anymore but only a fictitious force, invented to simplify the mathematics.

![Figure 1: A body acted on by a force at rest (a) and accelerated (b)](image)

In figure 2 it is assumed that the two balls of equal mass are fixed to the walls and mutually connected by a stretched spring. At rest we again have to state that 2 real forces $F$ of equal size and opposite direction are applied to each body.

![Figure 2: Two bodies, mutually connected by a stretched spring at rest (a) and rotating around their common centre of mass (b)](image)

If the 2 bodies are now set in motion in such a way that they rotate around their common centre of mass without a change in length and tension of the connecting spring, why are now the 2 forces to the outside fictitious and no longer real?

For students, especially for newcomers to the field of mechanics, it is quite obvious that the centrifugal force should be as equally real as the centripetal force and only rarely they follow the official explanation, where only one of these forces is declared as real and the other as fictitious. Such explanations disregard the early critique of Mach, published already in the 19th century, where he opposed to accept the fictitious character of forces of inertia. Most of all Mach pointed to the problematic concept of "absolute space", introduce by Newton to explain the existence of inertia and postulated a direct relation between the inertial mass here on earth and the mass of the stars out there in distant space.

The positions of Mach are debatable, especially with regard to the strict positivism he represents. Nevertheless the question is raised if it would not be more meaningful for physics teachers and
less confusing for their students to discuss the critics brought up by Mach and also his speculations about the origin of inertia and possible interaction partners than to stick rigidly to the distinction between real and fictitious forces.

2. Historical Review
Newton published his „Principia“ in 1687, where he postulated the existence of „absolute space“ as an essential basis to explain uniform and accelerated movements. When discussing his famous bucket experiment he asked: Against what the water shall accelerate and take on the parabolic surface if not against absolute space?
The existence of absolute space was quite early criticised by Leibniz and Berkeley but in the light of the overwhelming successes of Newton’s mechanics, especially in the field of astronomy, these critiques silenced for more than 150 years.
During this long time the existence of absolute space was generally accepted as a theoretical concept and not further questioned, while for practical reasons the fixed stars were used as the best accessible inertial system of reference.
The question, why especially the fixed stars do not move in respect to absolute space, remained open and had to be accepted as a coincidence.
In 1883 Mach published his basic critique of Newton’s mechanics. As strict positivist he opposed the pure theoretical concept of an absolute space and claimed that only relative movements and relative accelerations of interactive objects should be allowed to enter the laws of physics (Mach 1883).
Moreover he postulated a possible connection between the mass of the distant stars and the inertial mass here on earth. „Mass there influenced inertia here“ was his slogan.
In this way Mach laid the foundation of the modern development of mechanics which Einstein, profoundly influenced by Mach, then advanced first to the special and later the general theory of relativity.
For a more detailed description of Mach’s critiques of absolute space and his influence on Einstein’s work see Zylbersztajn (1994).
In his main book „The Science of Mechanics – a Critical and Historical account of its Development“ Mach wrote:
„The historical examination of the development process of a science is rather necessary if the collected sentences don’t risk to slowly transform to a system of semi-understood recipes or even to a system of prejudices.
The historical examination not only fosters the understanding of the existing but offers new possibilities by proving the existing as partly conventional and arbitrary.”(Mach 1960)
Many arguments can be brought up against Mach and his ideas. However the validity of this statement cannot be denied and may prove to be fruitful when applied to the way Newtonian mechanics is taught.

3. New developments
Einstein general relativity does not conform to Mach’s requirement that only relative terms should enter any theory of physics. Therefore many attempts have been made to implement Mach’s requirements in the frame of classical Euclidian space. Among these Schroedinger is certainly the most distinct researcher (Schroedinger 1925), when he tried to derive the equation for relativistic kinetic energy by adding a new term to the classical gravitational potential.
Recently Assis has presented an attempt where he claims to conform completely with Mach’s requirement for strict relativity of all physics terms (Assis 1999).
Assis goes back to a work of Wilhelm Weber, where Weber developed a basic law of electrodynamics as an extension of Coulomb’s law (Weber 1848).

\[ \vec{F}_{12} = \mathcal{H}_{e} \cdot q_{1} \cdot q_{2} \cdot \frac{\hat{r}_{12} \cdot \hat{r}_{12}}{r_{12}^{2}} \left( 1 - \frac{r_{12}^{2}}{2c^{2}} + \frac{r_{12} \cdot r_{12}^{*}}{c^{2}} \right) \]  

(1)

Assis applies an analog extension to the general law of gravity to arrive at the following expression:
Based on this force law he calculates the interaction between a central point mass and a rotating sphere, homogeneously filled with matter. Inserting the generally accepted cosmic values for radius and average density, the integral is taken over the whole universe and leads, among other results, to the well known Newton’s laws, however with a new interpretation. The system of the fixed stars is no longer by coincidence but by laws of nature the only and true inertial frame of reference.

Moreover inertia of a material body results as a gravitational interaction between the object here on earth and the masses of all the fixed stars in the universe.

4. Consequences for teaching physics

There are quite a few general results of Assis’ work and other researchers which partly contradict the theories of Einstein and which will not be considered further at this place.

In the light of these new ideas, however, the question can be raised if the traditional approach to teach Newton’s mechanics should be continued.

Why is the system of the distant galaxies a good inertial frame of reference? Is it correct to state that there is no interaction partner related to the forces of inertia?

Is this statement possibly as hypothetical as the statement, derived above, where the force of inertia is derived from an extended law of gravity?

If the answers to such questions would be discussed in a more open and speculative atmosphere and not as final results, students would be released from the rather difficult task to declare a force as fictitious which they continuously experience with their own body as real.

Moreover it could be that such a discussion would even come closer to what we call the truth.

To support such a discussion the simulation program Java-XYZ may be helpful where a special feature to visualize inertial forces has been implemented.

When ever meaningful the visualization of the inertial forces can be activated or deactivated as

```
\hat{F}_{12} = H_g \cdot m_1 \cdot m_2 \cdot \frac{r_{12}}{r_{12}^2} \left[ 1 - \frac{c^2}{c^2} \right]
```

An other example where the visualization of the inertial force – the centrifugal force - may be meaningful is shown in figure 4. Here the movement of 2 particle is shown, connected by an elastic spring and rotating around their common centre of mass.
These two examples are taken from a full introductory course for Newtonian mechanics which can be found under:
http://www.astrophysik.uni-kiel.de/~hhaertel/Mechanics_html/index.htm

The discussion of Assis’ work may also be helpful, when the students have to learn that there is a distinct difference between inertial and gravitational mass and that at the same time these two terms are strictly proportional.

A reference to Einstein’s theory of general relativity and the concept of a 4-dimensional curved space may be less helpful than a speculative discussion about the ideas of Mach and the results of Assis, who concluded that inertia here on earth is a consequence of a gravitational interaction with the distant galaxies in the whole universe.

References
Weber W 1848 Elektrodynamische Maßbestimmungen, Annalen der Physik und Chemie, 73: 193-240,
Mach E 1883 Die Mechanik in ihrer Entwicklung.
Mach E 1960 The Science of Mechanics, La Salle, The Open Court
Assis A 1999 Relational Mechanics, Apeiron Montreal

1 This book is available as pdf-file under:http://www.ifi.unicamp.br/~assis/wbooks.htm.