ANALYSING DIFFERENT FORMS OF PRESENTING NEWTON'S LAWS EMPHASYSING THE RELATED CONCEPTS

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1. Introduction

Our scientific knowledge is an organized conceptual structure that intends to capture some structural aspects of the world. In other words, we try to mirror a material system with a conceptual system.
In our scientific tradition we have as a model of conceptual systematization, Euclidian Geometry (300 B.C.); that is the first example of axiomatic systematization.
This particular conceptual system encompasses the geometrical knowledge accumulated through thousands of years of experience in land surveying, constructing and trading. Thus the axiomatization of a particular domain of knowledge is a late stage in the development of that domain. In such development the first step is the establishment of the pertinent concepts, that are usually decanted through years of controversy.
We have, for example, that the concepts of constant velocity and constant acceleration were established only with N. de Oresme and J. Buridan in the fourteenth century during a long controversy with the aristotelians. These thinkers also introduced through the theory of impetus, the concept of inertia that later was refined by Descartes, Galileo and Newton (H. Butterfield, 1957).
Once the pertinent concepts of a domain of knowledge are developed, these are used in statements about certain aspects of the world. In this way Galileo began to use the concepts of velocity, constant acceleration and inertia, to describe the motion of projectiles and falling bodies.
On these grounds, Newton structured the accumulated knowledge of the motion of bodies and tried to give this conceptual structure the format of Euclidian geometry, that is, the format of an axiomatized system.

2. Elements of an axiomatized system

Since Newton the idea is to give our physical theories the structure of an axiomatized system of propositions.
In such a system we distinguish:

1) Primitive concepts; these are concepts that are not explicitly defined within the conceptual system, as "point" or "line" in Euclidian geometry, or space and time in mechanics.
2) Definitions. Some concepts are defined in terms of primitive concepts or other definitions, as velocity and acceleration are defined in terms of space and time.
   Definitions are a matter of convenience, "but theoretically, all definitions are superflous"' (B. Russell, 1910).
3) Primitive propositions or axioms. These are propositions that are not proved within the system; they are rather the initial assumptions from which all other propositions follow by logical entailment.
4) Theorems. These are the propositions that can be proved starting from the axioms or from other theorems already proved; according to their importance, sometimes theorems are called lemmas or corolaries.

In the case of Newtonian Mechanics, as exposed in The Principia, we find some definitions, as that of mass, quantity of motion, and others, but there is not an explicit list of primitive concepts. However, Newton does state his three axioms or laws of motion. Hundreds of years later Poincaré would complain that Newton did not distinguish clearly what is experimental fact, what is a definition, or what is an assumption. Indeed, this complain is just an indication of the difficulties in axiomatizing a physical theory. Until now there is not a completely satisfactory axiomatization of classical mechanics, and this task is included in the fundamental problems listed by Hilbert.

A succesful axiomatic system must satisfy the following conditions:

I. It must be consistent, that is, a proposition and its negation can not be logical consequences of the system.

II. It must be complete, that is, any known true statement of the domain of knowledge that the system intends to mirror must appear either as a theorem or as an axiom.

III. The axioms must be independent, that is, an axiom cannot be a logical consequence of the other axioms.

Of course these conditions are not easy to establish and therefore most axiomatic systems are subject to permanent investigation. It was precisely the investigation of the independence of Euclid's axioms what lead to non-Euclidian geometries. These conditions are still harder to evaluate in physical theories.

3. Mach's Criticism to Newtonian Mechanics

As an example we consider Mach´s criticism to Newtonian mechanics. In this case we can see the influence of empiricism in Mach, who tried to reduce dynamics to kinematics, eliminating the concepts of mass and force because he saw in them traces of "metaphysical" concepts as matter (mass) and cause (force). A great deal of Mach´s view has passed to modern textbooks on mechanics. We give some examples in the appendix.

In his program Mach criticizes Newton´s definition of mass in terms of density and volume. However, since Mach does not list primitive concepts, his criticism is useless.

It is possible in principle to take density as a primitive concept and define mass in terms of it. There is some freedom to choose our primitive concepts as well as our axioms. The aim is to obtain a consistent and functional conceptual structure. Mach instead proposes to define mass in terms of accelerations. Here we find some problems since accelerations are vectors, while masses are scalars, however, it is commom to find this "definition" of mass in textbooks.

Besides Mach, following Kirchhoff, defines force as mass times acceleration. Thus Newton´s second law is regarded as superfluous and replaceable by a definition. This is again a common view in many contemporary texts. However, if force is really a definition then, following B. Russell, it is superfluous!

If the second law is not a proposition about the behavior of the world, but a definition, then when we find in a statement "force", we can substitute "ma", or vice versa. This cannot give us any information about the possible motions of a body.

Poincaré (1902) gives us some indications about this dilemma. He proposes that the acceleration of a body depends only of the position of the body and the positions and velocities of near bodies. This, he says, implies that the motions of any body are ruled by second order differential equations. If the world were such that velocities instead of acelerations depend on other bodies, as in Aristolian physics, then the equations would be first order, and if it were the case that change in acceleration depends on other bodies, then the equations would be third order. In this way we can see that Newton´s second law certainly is a statement about how the material bodies move under the influence of other bodies.

Hence our view is that it is much more sound to consider mass and force as primitive concepts than to pass them as useless pseudo definitions. Then the study of classical mechanics amounts to investigating the possible motions implied by the differential equation

$$\frac{d\vec{p}}{dt} = \vec{F}(\vec{r}, \dot{\vec{r}}, t)$$

for pertinent force functions F. In the study of these models of force, that the theory does not provide, lies the pragmatic aspect of classical mechanics (Moulines, 1979).
These force models may be suggested by experiment, like Hooke´s law, or may be conjectured, as the law of universal gravitation, but certainly they are additional hypotheses that must be fed into the general theory. Therefore the role of experience is to suggest and test our theories, but certainly it does not logically implies them.
Another point that deserves attention is the common statement that the first law of motion, the law of inertia, is a particular case of the second law: no force no acceleration. Sometimes it is said that the first law defines implicitly inertial frames of reference.
Here again Poincaré's observations may clarify the point. We can express the content of the first law as a further characterization of force: far away bodies cannot accelerate a given body. Hence the relation to inertial frames: the farther a body the better inertial frame it is. This is the reason why Newton proposed the "fixed stars" as the best inertial frame.

4. Conclusions

We can conclude that in teaching classical mechanics we must take into account that the involved concepts are "free creations of the spirit", as often remarked Einstein. Then the students must be acquainted with enough mechanical experiences in order to draw the pertinent concepts from the context of those experiences.
On the other hand, the laws relate these concepts and therein lies their usefulness to understand some aspects of the world. Thus our students must see that studying physics enables them to understand many phenomena and the technological advances familiar to us. In this respect it may be helpful to study the evolution of our knowledge in order to see how the concepts are created and refined. For example, Galileo saw the need to substitute the Aristotelian concept of resistance to motion by the concepts of inertia and friction, identifying this as a force.
This historical perspective can also warn us about the tentative character of our concepts. As Einstein once wrote: "And yet in the interest of science it is necessary over and over again to engage in the critique of these fundamental concepts, in order that we may not unconsciously be ruled by them." A. Einstein in the Foreword to "Concepts of Space" by , Max Jammer (1993).
Thus, in our view, the study of mechanics must begin with a familiarization with mechanical phenomena, as rotational inertia, motion of projectils, etc. Then we study how these phenomena have been understood in terms of different conceptual structures, as Aristotelian physics and Newtonian physics. The student must appreciate why Newtonian mechanics is a better approximation to mechanical phenomena than Aristotelian physics. This may be a beginning to a critical attitude that Einstein considers so necessary to the development of scientific knowledge.
As they advance in the study of mechanics, they can be exposed to the most elementary model of force, for example the almost constant gravitational force near the Earth. In advanced courses they may be put in contact with the mathematical structures of differential equations, variational principles and differential geometry, without forgetting that these structures give us an approximation to the motions that real bodies in interaction do have.

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