On the physics of partially static turbines

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In the past, there were many attempts to augment the power of wind turbines by means of diffuser systems or similar, but since these attempts did not respect energy- and momentum conservation, they have not been successful.

We present a first fundamental discussion of the physics of shrouded wind turbines. A design results, which is different from the diffuser approach – the partially static turbine. It consists of a rotating propeller and of a static element, which allows the propeller to extract energy from an external flow of fluid, which does not pass through the propeller itself.

Introduction

Betz has shown, that a wind turbine of propeller radius r can extract from a flow of wind at maximum 59% of the energy of the free flow through an area equal to the area covered by the rotor, \( r^2 \pi \) \(^{(1,2)}\). There have been many attempts to increase the power of a wind turbine beyond this limit by means of diffuser systems \(^{(3-7)}\), but these attempts have never resulted in practical applications.

This failure might be attributed to the fact, that a fundamental physics discussion – as it was done by Betz for the classical wind turbine – does not yet exist for wind turbines with a shroud system. For instance, it is unknown, by how much the velocity of the air passing through the turbine can be increased by a conventional diffuser system and by how much the power of the turbine can be increased.

As a first step we will derive in this paper the Betz limit, proceeding in a somewhat different way as has been originally done by Betz. Our somewhat more general arguments will allow us to then determine, whether a diffuser system can in principle augment the power of a wind turbine. We will find, that it cannot. We will then discuss a possibility to overcome the shortcomings of conventional diffuser systems – the partially static turbine.

The feasibility of such a system was qualitatively studied in \(^{(8)}\), and some first experimental results have been presented in \(^{(9)}\).

The emphasis of this paper is to discuss the mode of functioning of diffuser systems and partially static turbines from a fundamental physics point of view. Fluid dynamic model calculations will only serve to illustrate the physics discussion.

Unshrouded turbines

\(^{(1)}\) To be published in Renewable Energy, Elsevier.
We assume a flow of air at atmospheric pressure and velocity $v_1$. When this flow approaches a conventional turbine, its velocity decreases and its static pressure increases correspondingly (following the law of Bernoulli). The air passes through the turbine at velocity $v_2$. The turbine is thought to be a thin disc, which causes a pressure drop in the flow, $\Delta P$. After having passed the turbine, the flow continues to slow down, thereby increasing its pressure again, until it finally reaches a velocity $v_3$ at atmospheric pressure. We assume the turbine to be ideally efficient. This is as described in (1).

When a volume of air with length $\Delta r$ and section $S$ passes through the turbine, its energy gets reduced by $\Delta E = F \cdot \Delta r^2$, where $F = \Delta P \cdot S$ is the force, which acts on the area $S$ of the turbine. This energy gets transferred to the turbine (energy conservation, and assumption of the turbine to be ideally efficient). This same force also causes a transfer of linear momentum to the turbine $\Delta p = F \cdot \Delta t$, where the time $\Delta t$ is the time, which is needed by the volume of air to pass through the turbine at velocity $v_2$, so that $\Delta t = \Delta r / v_2$.

The energy- and momentum transfer to the turbine therefore gives us:

$$\frac{\Delta E}{\Delta p} = \frac{F \cdot \Delta r}{F \cdot \Delta t} = v_2$$  \hspace{1cm} (1)

The difference in energy and momentum of the volume of air of mass $m$ before interacting and after having interacted with the turbine, is:

$$\Delta E = \frac{1}{2} m (v_1^2 - v_3^2)$$  \hspace{1cm} (2)

and:

$$\Delta p = m (v_1 - v_3)$$  \hspace{1cm} (3)

inserting equations (2) and (3) in (1) yields: $v_2 = \frac{\Delta E}{\Delta p} = \frac{1}{2} (v_1 + v_3)$, \hspace{1cm} (4) \hspace{1cm} in agreement with (1).

As explained in (1), the power of the turbine is maximum when $v_2 = \frac{2}{3} v_1$, and the corresponding power is the above mentioned limit of Betz.

**Diffuser**

Water turbines often have a diffuser at their outlet. In the diffuser the velocity of the water decreases, and its static pressure increases correspondingly, so that the pressure drop in the turbine (and therefore the power of the turbine) will increase – a part of the kinetic energy of the water, which leaves the turbine, and which otherwise would be lost for energy production, is recovered by the diffuser. The energy density of the water does not change, when flowing through the diffuser.

\[\text{As is done in the original work of Betz, it is implicitly understood throughout our discussion, that for instance } F \cdot \Delta r \text{ or } F \cdot v \text{ are the scalar products of the vectors } \vec{F}, \vec{v} \text{ and } \Delta \vec{r}.\]
For a water turbine equations (1) and (3) are still valid, but equation (2) must be substituted by
\[ \Delta E = \frac{1}{2} m(v_1^2 - v_3^2) + E_p, \]
where \( E_p \) is the energy of a volume \( V \) of water, under the pressure \( P \).

Equation (4) then becomes:
\[ \frac{\Delta E}{\Delta p} = v_2 = \frac{1}{2} (v_1 + v_3) + \frac{E_p}{m(v_1 - v_3)} = \alpha \cdot \frac{1}{2} (v_1 + v_3) \quad \text{with} \quad \alpha = \frac{\frac{1}{2} m(v_1^2 - v_3^2) + E_p}{\frac{1}{2} m(v_1^2 - v_3^2)} \quad (5) \]

For water turbines, \( \alpha \) can be much larger than 1, and therefore \( v_2 \) can become much larger than \( \frac{1}{2} (v_1 + v_3) \). But for wind turbines \( \alpha \) is equal to 1.

A different way of phrasing the situation is as follows: let us assume, that air passes a turbine at velocity \( v_2 \), causing a force \( F \) on the turbine, thereby transferring energy and momentum to the turbine \( \Delta E = F \cdot \Delta t \), \( \Delta p = F \cdot \Delta t \), where \( \Delta t / \Delta t = v_2 \), as described above. If we now increase the velocity \( v_2 \) (let us assume for simplicity, that \( F \) remains unchanged), the energy transferred per \( m^3 \) of air will not change, since the value of “\( \Delta r \)” in \( \Delta E = F \cdot \Delta r \) does not change. But since the air will pass now in a shorter time interval \( \Delta t \), the momentum transferred will be smaller than before. Therefore from energy conservation one would conclude, that the air should again have the same velocity \( v_3 \) after having passed the turbine. But from momentum conservation one would calculate a higher value for \( v_3 \). Therefore, the assumption that \( v_2 \) could be larger than \( \frac{1}{2} (v_1 + v_3) \) must be wrong. And since in (1) “\( F \)” cancels, the argument holds for any value of \( F \).

As a consequence, it is not possible to augment the power of a wind turbine beyond the Betz limit by means of a conventional diffuser. Note, that in a conventional diffuser system the flow through the turbine and the flow through the diffuser are approximately the same.

**Partially static turbines**

We next assume a flow of air, which is subject to two forces: a force \( F_a \) extracts energy from the flow at a mean velocity \( v_a \), and a force \( F_b \) extracts energy from this flow at a mean velocity \( v_b \). The energy and the momentum which are extracted from the total flow in a time interval \( \Delta t \) are then:
\[ \Delta E = F_a \cdot v_a \cdot \Delta t + F_b \cdot v_b \cdot \Delta t \quad \text{and} \quad \Delta p = F_a \cdot \Delta t + F_b \cdot \Delta t. \]

Applying equation (1) to this situation and with equations (2) and (3) we get
\[ \frac{\Delta E}{\Delta p} = \frac{(F_a v_a + F_b v_b)}{F_a + F_b} = v_2 \quad \text{(where \( v_2 \) is again defined as \( v_2 = \frac{1}{2} (v_1 + v_3) \)) or:} \]
\[ F_a \cdot v_a + F_b \cdot v_b = F_a \cdot v_2 + F_b \cdot v_2 \quad (6) \]

Obviously, this requirement can be met for \( v_a = v_b = v_2 \). But it can also be met for \( v_a > v_2 \): a wind turbine may extract energy from a flow of air at a velocity larger than \( \frac{1}{2} (v_1 + v_3) \), if there is an additional energy extraction from the flow at a smaller velocity.

If the two forces \( F_a \) and \( F_b \) both act on the same flow, then we see from formula (6) that none of them can extract power in excess of the power, which is extracted by one combined force \( F_a + F_b \).
acting on the flow at the velocity \( v_2 \). This situation would be represented for instance by two turbines one behind the other, acting both on the same flow – obviously none of them can extract more energy than what could be extracted by one single turbine of good efficiency. This situation would also be represented by a turbine with a diffuser, if both act on one and the same flow of air. Again, the shrouded turbine could not extract more energy than an unshrouded turbine. If instead the total flow is composed of two neighboring partial flows and the forces \( F_a \) and \( F_b \) act on each of these flows, then one of them may extract power in excess of what could be obtained without the second force and the additional flow being present. This remains also true, if the two forces act on different spatial regions of one flow in different ways. Of course, for this to function, there must be an exchange of momentum and energy between the two flows. This should however not be too restrictive a requirement – momentum and energy exchanges between two flows are a quite common phenomenon in fluid dynamics, examples are to be found in the wake of a jet engine or in a jet pump.

It should therefore be possible to realize the situation described in equation (6) by means of a propeller, which extracts energy from a flow of air under the force \( F_a \) at a large velocity \( v_a \), (so that less momentum is extracted than what would correspond to the energy transfer) and a shroud system, which acts also on an additional flow of air and which provides an additional force, correcting the energy-momentum balance for the entire flow.

The difference between a diffuser system and a partially static turbine is therefore twofold: First, in a diffuser the flow obeys the law of Bernoulli, the change in velocity is due to a change in cross section only, and therefore the energy density of the fluid is constant (apart from losses in friction or turbulences). Instead, in a partially static turbine there is an additional force acting on the fluid, a force which cannot be accounted for by the equation of Bernoulli, and which is directed upstream. Secondly, in a partially static turbine, there is apart from the flow through the area which is covered by the rotating propeller a significant additional flow, which is absent in a diffuser system.

In the following, these arguments will be discussed in some more detail, based on a fluid dynamic model simulation.

We note, that the discussion presented in this chapter gives a first approximation only for at least two reasons: first, we have assumed the validity of the disc model for describing the turbine. Second, we have not considered the action on a shroud system of the flow which is passing at its outside. For getting a more detailed description of partially static turbines, both arguments need to be studied in more detail in future investigations.

**Fluid dynamic simulation**

We simulate a wind turbine by means of the fluid dynamic simulation program Star-CD, which is a standard in many industrial applications. Note, that in the context of this paper we are using this program only for the purpose of demonstration. A fundamental scientific study from a fluid dynamic point of view is left to a future investigation. In order to save computing resources, the propeller of the turbine is very simple, it consists of rectangular two-dimensional propeller blades. The simulation volume is subdivided in about 40,000 cells. In the azimuthal coordinate \( \phi \), only 10 degrees of the model are simulated, cyclic conditions are imposed on the side walls. Wind enters along the x-axis at a velocity of 5 m/s, the outer walls of the model are kept at ambient pressure. From the velocity of the air passing through the turbine (referred to as \( v_2 \) in the last chapters) and from the force acting on the propeller, \( F \), we can calculate the “available power” \( P = F \cdot v_2 \). Since
we are now dealing with a non-ideal turbine, not all of this power can be used: from the angular momentum on the blades, $L$, and from the speed of rotation, $\omega$, we can determine the power of this turbine $P = L \cdot \omega$. We will refer to it as “useful power”.

The propeller of the turbine has a radius of 56 cm and therefore covers an area of $0.56^2 \pi \ m^2 = 1.0 \ m^2$. A flow of air at $v = 5 \ m/s$ has a kinetic energy of 15 J/m$^3$ and a power of 75 W/m$^2$. In the Betz limit, an ideal turbine could absorb a power of $75 \cdot 0.59 \ W = 44W$, when exposed to this flow.

In figure 1 we show the useful power of the turbine as a function of its speed of rotation in this flow of wind at 5 m/s. The maximum of the power is reached at 21 W and at $\omega = 125$ rotations per minute about. At this rotational velocity we find $v_f = 4.2 \ m/s$ (averaged over the area covered by the rotor). From this observed value and from equation (4) we determine a value of $v_f = 3.4 \ m/s$. That means, that the propeller extracts somewhat more than half of the kinetic energy of the air. The propeller blades are exposed to a force in the $x$-direction of $F = 7.4 \ N$, corresponding to an available power of 31 W.

Figure 2 shows a cross section through the model, along the $x$-axis. Air is entering from the left. The static pressure in the device is shown in steps of 3 Pascal. There is an increase in pressure before the turbine, a sudden drop in pressure when the air passes through the propeller, and then again an increase in pressure.

Next, this propeller is inserted into a cylindrical shroud, which has a profile similar to a sail. The configuration is shown in figure 3, which again shows a cross section along the $x$-axis. The inner radius of the shroud at its narrowest section is 0.76 m. Where the shroud is widest, its radius is 1.3 m. The shroud is 1.1 m long.

The static pressure field shown in this figure is on the same scale as in figure 2. At the inlet of the shroud, and also in front of the turbine, the static pressure has become negative, the velocity of the air there is larger than 5 m/s.

Again in figure 1 we show the (useful) power of the partially static turbine. The power has increased from 21 W to 108 W, and its maximum has shifted from 125 to 270 rpm.

In table 1 we compare some parameters of the bare turbine and the same turbine with a shroud system.

The useful power of the turbine has increased by a factor of 5.2, the available power by a factor of 4.3. The useful (available) power is above the Betz limit by a factor of 2.4 (3.0).

At a velocity of 5 m/s the undisturbed airflow has a power of 75 W/m$^2$. Since the velocity of the airflow through the turbine is increased to 7.9 m/s, the power of the flow through the turbine correspondingly is 119 W, which is less than the available power of 135 W. This difference of $(135-119)W = 16 \ W$ is only a part of the energy transfer, which must occur from the external to the internal flow:

The force on the propeller of 17 N (referred to as $F_a$ in the previous chapter) causes a momentum transfer of 17 Ns. Since in each second 7.9 m$^3$ of air flow through the propeller, the resulting momentum loss for each m$^3$ of air is 17/7.9 Ns = 2.1 Ns. We next need to consider : the free flow has a linear momentum of 6 Ns/m$^3$, and the velocity of the air (which has passed through the turbine) a few meters behind the turbine is somewhat less than 1 m/s, corresponding to a linear momentum of less then 1.2 Ns. As a consequence, the air looses a total of 4.8 Ns, when travelling though the system, but only 2.1 Ns are extracted by the action of the propeller. The remaining momemtum transfer of 2.7 Ns must be attributed to the shroud system. (And this extraction of linear momentum
from the inner flow must again be accompanied by a respective energy transfer, which in turn must be compensated for from the energy of the outer flow).

Figure 4 shows the absolute pressure in the system. The figure illustrates, that indeed the shroud is not just an external device, merely changing the velocity of the air flow (like a conventional diffuser does), but rather is participating itself in the process of energy extraction. The static shroud and the rotating blades form together an energy extracting system, a turbine. Hence the name “partially static turbine”.

Though we have simulated a wind turbine, the same physics arguments also apply to water turbines. With the method discussed in this paper one can also extract energy from a flow of water of large cross section with a turbine of much smaller cross section. The water level at the inlet of the device does not have to but may be above the one at the outlet, so that it becomes possible to make use also of very small differences in water level, not accessible to present technology.

Conclusions
A conventional diffuser system, as it is commonly used in water turbines, cannot augment the power of a wind turbine beyond the Betz limit for reasons of energy- and momentum conservation. If instead the propeller of the turbine is embedded into an external flow of air, from which by means of a static structure energy can be transferred to the internal flow (the flow through the propeller), then the propeller can supersede the Betz limit with respect to this internal flow.

In a simulated model we have increased the power of a wind turbine by a factor of 5 by means of an external shroud system. This increase by a factor of 5 does not represent a physical limit. We expect, that the power of partially static turbines will finally be limited by the formation of turbulences. This needs to be clarified in future research.

References
3. A.Betz, (Energieumsetzung in Venturieduesen. Naturwissenschaften 10, pp.160–164, (1929)), we only know about the existence of this article, which suggests that already Betz has attempted to exceed the Betz limit, we could however not find a copy of this article.


10. For a description of Star-CD see [www.cd-adapco.com](http://www.cd-adapco.com) and the references there.
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<th>Bare propeller</th>
<th>Shrouded propeller</th>
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<tr>
<td>$v_2$ 4 m/s</td>
<td>4.20 m/s</td>
<td>7.95 m/s</td>
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<tr>
<td>$F$ 7.45 N</td>
<td>16.9 N</td>
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<tr>
<td>Available power 31.1 W</td>
<td>135 W</td>
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<td>Useful power 20.9 W</td>
<td>108 W</td>
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Table 1: Comparison between bare propeller and the same propeller with an external shroud system.

Figure 1: Power of bare propeller and partially static turbine as a function of speed of rotation.
Figure 2: Static pressure around the turbine. Wind enters from the left. The scale is in steps of 3 Pa.
Figure 3: Static pressure in a partially static turbine. The scale is as in figure 2.
Figure 4: Absolute pressure in the partially static turbine in steps of 2 Pa.