Reasoning and Models of Talented Students on Electrical Transport in Solids

Giuseppe Fera, Physics Education Research Unit, University of Udine, Italy
Marisa Michelini, Physics Education Research Unit, University of Udine, Italy
Stefano Vercellati, Physics Education Research Unit, University of Udine, Italy

Abstract

A sample of 40 students, selected from the last two classes of Italian secondary schools to participate in a school of excellence in modern physics, after measuring resistance as a function of temperature of metals and semiconductors and Hall coefficient at room temperature, was engaged in conceptual exploration of contexts related to the nature of charge, voltage, current and electrical resistance. Results highlight that the construction of the connection between macroscopic phenomenology and microscopic models concerning electrodynamics processes is not only a success in physics, but it is also a possible way to address the widespread and persistent difficulties that students face during the building of the interpretative models.

1. Introduction

In the all school levels (primary, secondary and university) the topics of electrodynamics are in the curricular plans among all the university institution across Europe (Euridyce, 2011).

Research literature highlight the presence of persisting learning knots also in students of secondary schools as concern charge, electric field, potential and the meaning of conductivity (Duit, R. & von Rhôneck, 1998; Mulhall, McKittrick, Gunstone, 2001).

The understanding of the phenomena is closely related to models, analogies, simulations that students hardly integrate into a coherent and comprehensive conceptual framework (Stocklmayer & Treagust, 1996; Wittman, Steinberg, Redish, 2001).

In literature, a wide discussion address the opportunity to adopt a microscopic approach to the phenomenology of electrical conduction (Eylon & Ganiel, 1990; Psillos, 1998; Thacker, Ganiel & Boys, 1999; Chabay & Sherwood, 1999).

In particular, recent works shows how the understanding of the relationship between the physics quantity involved and the electrodynamics phenomenology requires the clarification of the relationship between charge, current, voltage and electric field (Hirvonen 2007; Hart, 2008; Stocklmayer, 2010).

The learning, considered as the conceptual knowledge of these relations, state the problem of the nature of the quantity themselves and in particular the overcoming of the following aspects: 1) conceptual connection between electrostatics and electrodynamics, 2) the role of the electric field and its dynamic relationship with the electric charge, 3) the overcoming of the idea that relations between charges and electric field are local. The addressing of a microscopic model of electrodynamics arises therefore in these terms and not in terms of possible simple descriptive mechanisms or figurative representations useful to memory.

Research question are:

1. what are the conceptual referents of students’ reasoning on the electrodynamical phenomena and how they play an interpretative role?
2. how different activities (simulations, experiments, analogies) promote the integrated interpretation on macro / micro levels and how these can be used in the different contexts?
3. what interpretative role plays a quantum description in continuity with the classical model?
2. Context and method

The experimental intervention was proposed during the Summer School National Modern Physics (Udine, July 2011) organized as part of the project IDIFO3 of the National Plan for Scientific Degrees. It was addressed to 40 participating students selected on the basis of grades in physics from Italian students enrolled in the fourth and fifth classes of upper secondary school (grade 11th and 12th).

The learning path is divided into two phases: Activity 1) a laboratorial exploration of the electrical properties of solids, Activity 2) a seminar concerning models of electrical conduction in solids.

Activity 1: Students perform measurements of resistivity of semiconductors, metals and superconductors with respect to temperature from liquid nitrogen temperature to room temperature and measure the sign and the density of the charge carriers by Hall effect in copper zinc and semiconductors at room temperature.

Activity 2: during the seminar were addressed: the nature of charge, potential, current and electrical resistance; the electrical conduction processes in solids and the dependence of the resistance of metals and semiconductors versus temperature.

The Focus of Activity 1 is on the students’ Lab work for the characterization of the resistivity in the metals and semiconductor samples; in Activity 2 on the conceptual reconstruction of the students’ interpretation of the experimental data on the light of the related learning nuclei.

The students’ reasoning and the ideas on each specific topic of the seminar were collected using personal worksheets from each student.

The analysis of the data collected was done in a qualitative way, organizing the interpretations in profiles of reasoning and different uses of the concepts (Groves et al., 2004).

Activity 1 – Laboratorial exploration

Using USB probe for resistivity versus temperature and Hall coefficient measurements (Gervasio & Michelini, 2009) students quantitatively explore the electrical properties of metals and semiconductors. The measurements of resistivity and Hall effect allow them to obtain information on the sign of charge $q$, concentration $n$ and mobility $\mu$ of charge carriers.

![Diagram of Hall effect](image)

**Figure 1.** Scheme of Hall effect

Defining the Hall coefficient

$$R_H = c V_H / (l B)$$

and measuring $c$, $V_H$, $B$, $l$ (fig. 1) we find

$$qn = 1/R_H$$

Measuring the resistivity $\rho$, it is possible obtain the mobility

$$\mu = R_H / \rho$$
For example, using a sample with $c=0.9\pm0.05$ mm and $B=290\pm5$ G and assuming $q$ equal to the electron charge, students interpolating data as in fig. 4, find the concentration $n=10^{20}/m^3$ of the charge carriers in Ge::P (p-doped Ge) and the sign of the charge result to be positive.

**Figure 2.** Copper resistance versus temperature

**Figure 3.** Ge::P resistance versus temperature
Figure 4. Hall effect in Ge::P

Figure 5. Hall effect in Zn

Activity 2 – Seminar

A seminar discussion in form of interactive demonstration (Sokoloff et al., 2007) was held with tutorials cards and monitoring of reasoning. Following topics are addressed:

1. Electrification. The electrostatic phenomenology justify the assumption of the charge carriers in the material and the concept of potential difference as the engine of charge transfer (Mossenta e Miche- lini, 2010)

2. Current and resistance. The analysis of measures of voltage versus current in circuits with battery, bulb and wires of different material, length and cross section founds the concept of electrical resistance of a wire

3. Dependence of the resistance versus the temperature. The change of the brightness of the bulb while the circuit is submerged of liquid nitrogen recall the trend of resistivity versus temperature of conductors such as Cu and Zn and semiconductors

WCPE 2012, Istanbul, Turkey
4. A microscopic mechanistic model of free electron gas in metals is discussed by Supercomet simulation (http://online.supercomet.no/). In this simulation the motion of negative charges is represented in a lattice of positive ions. In the absence of voltage applied across the wire, the motion of negative charges is random and their displacement on average is zero. By applying a voltage across the wire ordered motion overlaps the random motion and it is detected as current. The (constant) velocity of the ordered motion is called drift velocity. Increasing the temperature of the wire, the oscillations of the positive ions around the equilibrium positions increase in amplitude. Conceptual issues of the Drude model is introduced to students with help of this simulation and it was used to predict the evolution of the resistivity of a metal with the temperature; added values and limits of model were analyzed.

5. The energy band model. Piekala chair energy levels (Golab-Meyer, 1991) is used as starting point of a discussion on the energy levels model of the electronic states and to construct the energy band model by means of the simulation proposed by Dean Zollman (http://phys.educ.ksu.edu/vqm/html/eband.html). The quantum-mechanical model is used to interpret the experimental situations already examined by the students.

The core: overcoming of the classical model

Using the classical model (Activity 2, step 4) shown by Supercomet simulation, students interpret the values of resistivity of many metal conductors, but the Hall effect (fig. 4 and 5) highlights also that the sign of the charge carries in metal could be also positive (as for zinc). This result cannot be justified in the context of the free electron gas model.

The electrical conductivity of semiconductors (fig. 3) is many orders of magnitude smaller than metals. Therefore, the concentration of free carriers in semiconductors is many orders of magnitude below the average values of metals. At low temperatures the behavior of the resistivity of semiconductors is qualitatively similar to the metals (fig. 2): resistance increases with temperature through an almost linear law. On the contrary, the behavior at higher temperatures clearly indicates the need for a new physical interpretation. The quick fall of the resistance shows an exponential dependence of the concentration of free carriers from the temperature. This means that the concentration of free carriers increases with temperature in such a way proportional to the factor $\exp(-A/kT)$: this result is consistent with the hypothesis of a mechanism of electrical conduction thermally activated by overcoming a “energy barrier”. These experimental facts require a major change in the microscopic model, which can no longer be built ignoring quantum physics. The free electron model becomes a only intuitive reference, that leaves room for a representation based on assumptions centered on state properties of the carriers.

3. Data and Data Analysis

We discuss here the data referred to Activity 2 step 3, 4 and 5.

As concern step 3, students were asked to predict the change in the brightness of the light bulb when the temperature of the wire is lowered. In agreement with the findings of Wittmann et al. (2002) related to the microscopic processes in the wire, a 5% of students expected that the brightness decreases because “in copper wire cooled electrons moving more slowly”. Furthermore, the increasing of brightness is interpreted by 48% of students with the motivation: “ions of the lattice vibrating less, they hindering less the motion of the electrons”.

Students who answered use the following argumentation in support of their predictions: the thermal agitation increases and so the collision/obstacles for the moving charges increase (43%), more resistant at room temperature (8%), at room temperature the motion of the charges is more disordered (8%), at room temperature the electrons are slowed down by the resistance (8%) at room temperature the particles are more excited (3%), the nitrogen liquid facilitates the transmission of electricity (3%), at room temperature the bonds between electron and nuclei are stronger (3%).

As concern step 4, this qualitative interpretation in terms of “kinetic electron gas in a lattice of ions” is also used by students to explain the observed trend of the resistivity in the copper versus the temperature.
The graph of resistivity versus temperature of semiconductor Ge::P, which have a completely different trend, highlight the limitations of the model but only for 8% of students.

Then the use of free electrons model was deepened for the conduction in metals using the Supercomet simulation. 38% of students recognize that at higher temperatures the thermal agitation of the lattice ions is higher and explains the observed trend in the copper resistance with temperature, but, between them, only 60% relates the increase of temperature with a higher frequency collision with the ion lattice. The 35% recognizes that the model cannot be used to explain the changes in the resistivity of the semiconductor with the temperature.

The analysis of the plausibility of the physical properties of charged particles represented in the simulation, sees difficulty in 60% of the students on the particle size and in 70% of the students on the speed of the moving charges. Some alternative conceptions reported by other authors (Wittmann et al., 2002, De Posada, 1997) emerge among the replies: the electrons of conduction are subject to the Coulomb force (3%); the drift velocity is much larger than the one of the random motion (10%).

As concern step 5, students describe the states that the chair can have in terms of potential energy to the floor: 35% recognize the change of the potential energy of the chair, 13% recognize the presence of discrete energy levels of the chair, 5% recognize the change of position of the chair.

The state of a physical system can be described by means of a representation of its energy levels, which can be discrete. This is the case of a ball in a bowl. Only 9% of students classifies the states that the ball can take with respect to its energy.

The use of energy band model to interpret the electrical conduction in metals highlights some not-common alternative conceptions: 8% of the students explain the difference between metals and insulators in terms of the width of the gap, 8% do not distinguish the promotion of electrons to high energy level from drift motion, 3% introduce in the model the gravitational interaction and the attraction between the ions of the lattice, while, indeed, the 36% use the model according physical vision. However, the energy band model manifests its explanatory capacity in relation to electrical conduction in the intrinsic semiconductor, in particular as regards the trend observed in resistivity as a function of temperature (descending in the intrinsic zone). Applying the model in this context, 25% of students do not recognize the role of the amplitude of the gap, 13% correctly describes the conduction in the intrinsic zone, but only 5% correctly uses the model to distinguish intrinsic from extrinsic conduction.

4. Conclusions

As concern RQ1, students use macroscopic Ohm’s laws as a conceptual referent to analyze the different current intensity as a function of length / section / material of the wires. They evoke microscopic interpretations, but only 15% refers with awareness the existence of a disordered motion of electrons in a wire at room temperature. The concept of resistance is most often associated with an intuitive idea of opposition to the movement. The presence of these ideas indicates that the reflection at the microscopic level is not activated before the use of the Supercomet simulation.

As concern RQ2, the larger fraction of replies leads back the variation of the resistance of a metal with the temperature to the motion of thermal agitation of the lattice ions, confirming so the validity of the Supercomet simulation. There are some limitations as concern the representation of the correct relationships between the physical properties of particles and the unsuitability to describe the behavior of semiconductors. However, the Supercomet simulation seems to offer students an effective tool for representation of the microscopic world, but there is a need to critically discuss in detail the following aspects: dimensions of electrons; dimensions of atoms, lattice pitch (average distance between two ions reticular first neighbors); concentration of conduction electrons; average speed of conduction electrons; mean free path of the electrons of conduction (average path length of an electron between an interaction with a lattice ion and the next).

As concern RQ3, the introduction of discrete levels using the analogy with the energy levels of a chair helps to overcome the difficulties related to the energy levels model and provides a conceptual tool that all students take to approach the interpretation based on the band structure. This conceptual tool is the basis of the formalism and provides a complete description of the observed phenomena.
More than one third of the students interpret the phenomenology of electrical conduction in metals according to the energy band model; the percentage falls drastically to 5% when they interpret the dependence of the resistance of semiconductor with the temperature in different regions of intrinsic and extrinsic conduction. The complexity of the phenomenon requires to recognize and evaluate the contributions by different physical processes in a global view which connects microscopic models and macroscopic quantities: our efforts continue in this direction.

References


Eurydice, Recommended annual taught time in full-time compulsory education in Europe, online http://eacea.ec.europa.eu/education/eurydice/


