CoLoS IDEAS APPLIED TO THE STUDY OF A VACUUM TUBE DIODE

Ernesto Martín, José M. Zamarro, *University of Murcia*

Abstract
A simulation of the vacuum tube diode with plane cathode and anode is presented. A study of the (2D classical) thermionic emission taking place at the cathode is carried out in terms of a gas. The characteristic curve of the diode (current vs. applied voltage) is investigated using the simulation in the space charge regime. This simulation is based on the interaction of the emitted electrons with the applied electric field and with the field of the other emitted electrons. In this way, two problems usually studied in terms of statistical physics and advanced electromagnetic partial differential equations are studied in terms of some few well known basic laws. This approach is at the core of the CoLoS way of teaching.

1. Introduction
Thermionic emission is the process taking place in hot metals where the electrons can achieve enough energy to surpass the confining extraction barrier. A simple model for a metallic conductor considers the system as a potential well where the electrons can freely move but cannot get out. At room temperature, electrons in a metal are prevented from escaping through its surface due to a potential energy barrier (work function energy), that the electrons can surpass if the metal is heated enough. To model this process we have considered a classical gas confined into a box where the particles can go out when they are close to the surface and get kinetic energy (associated to the velocity component perpendicular to that surface) greater than the specified work function of the metal. This model does not correspond to the real system because we deal with particles obeying classical statistical laws (instead of confined electrons subjected to Fermi-Dirac statistics). This not real model results in a change of the Richardson Dushman (Owen 1929) law for the emitted current, that can be analytically justified in terms of the differences above mentioned. For this work we have reworked the obtaining of the corresponding Richardson Dushman law in terms of the proposed model and describe the results obtained with the simulation, which totally agree with the theory.
In addition, we have modeled the process taking place in a vacuum tube diode, composed of two parallel plate electrodes, the thermionic emitting cathode and the anode connected to a given potential in respect to the cathode, under space charge regime. To do this we have considered the electrons emitted by the cathode and flying to the anode as electrically interacting through Coulomb's Law and additionally subjected to the externally applied electric field associated to the anode-cathode potential difference. In a simple model, where we have not imposed the constancy in the potential of the two metallic electrodes, a similar behavior to the Child Langmuir law (Child 1911) is obtained, showing the saturation behavior when the anode applied voltage is large enough.
All simulations have been developed with Ejs *(Easy Java Simulations)* (Ejs 2009), an authoring tool developed by F. Esquembre from the CoLoS *(Conceptual Learning of Science)* Murcia group, that facilitates the work to people not experts in programming.

2. Thermionic Emission Simulation
Two simulations has been developed, the first one simulates a two dimensional ideal gas to study the thermionic emission, the other simulation studies the movement of electrons between the plates of the diode.
In figure 1 we can see the interface of the thermionic emission simulation. In the left part of the main window eight hundred particles (this number can be varied at wish) collides freely, when a particle arrives to the right side of the left rectangle with energy greater than the work function it escapes. In the upper part there are some typical control buttons and the energy of the gas is shown. In other window we can select the work function and the temperature through the *K*T* factor.
The current density is calculated by counting the particles flowing per unit time. In this way a study of the relation between the temperature and the current density was accomplished.

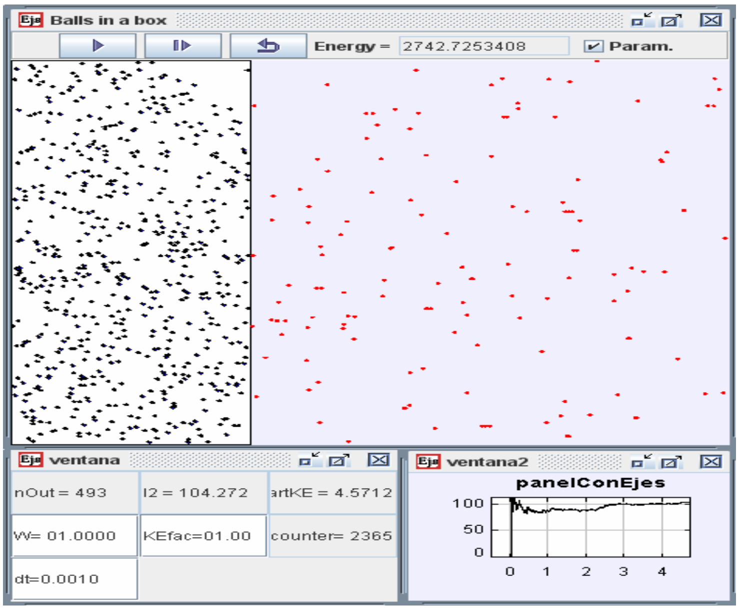

Figure 1: Simulating the thermionic emission. Heated metal modeled as a gas of particles (left). Emitted particles are also shown (right).

3. Testing Richardson Law

We have used the previous simulation to test the relationship between the current density and two parameters the work function and the temperature. Hypothesis: The Richardson like equation for our system has the form

$$J = f(T)\ \exp(-W/kT) \qquad (1)$$

where J is the current density, T is the absolute temperature, W the work function and k the Boltzmann constant.

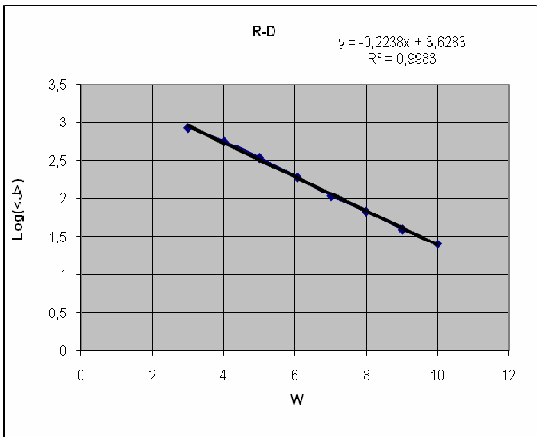

Figure 2. Fitting the values of log(J) versus work function W

Analyzing the current density for different values of W keeping the total energy (proportional to T) constant, we can check the exponential dependence by recording $log(J)$ as a function of W. From the fitting, a value of kT can be obtained and then the total energy kTN of the gas. In the case shown, a value of 2681 was obtained. This can be compared to the total kinetic energy of the gas: 2680 in our simulated experiment.

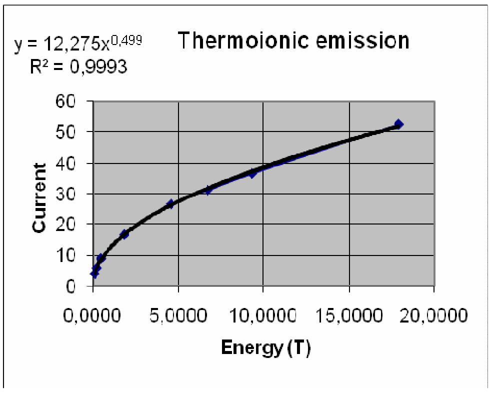

Figure 3. Values of density current, J, versus energy W

Other important fact of this equation is the factor *f(T)* that we can also investigate with the simulation. The results recorded in Fig. 3 correspond to different situations where we have changed the Temperature keeping the ratio (*W/kT*) constant. This allow us to investigate *f(T)* in equation 1. Fitting a potential curve to our data results in

$$f(T) = A\,T^{1/2}. \qquad (2)$$

If the principles to obtain the Richardson Dushman (Hemenway 1962) equation are applied to our model, a two dimensional ideal gas and the Boltzmann distribution instead of the Fermi one, we analytically obtain a dependence in the form $T^{1/2}$ for the studied factor. This dependence corresponds to the one obtained with our simulation. Interesting to note that this dependence was the first one that Richardson proposed in 1901 (Owen 1929). It was not until 1911 (Owen 1929) when he concluded in the presently accepted thermionic emission law:

$$J = AT^2 \exp(-W/kT) \qquad (3)$$

4. Current – Voltage Relationship in a Diode

We have analyzed a diode formed by a cathode and an anode closely spaced and formed by parallel plane surfaces. In figure 4 we can see the interface of the simulation of a two dimensional diode. Our model is as follow, an electric field is applied between the two plane electrodes while a constant rate of particles per unit time is provided to the space between plates, emitted by the cathode. The movement of a particle is the result of the action of the applied electric field and the interaction with the other charged particles, this result in a current flow between plates.

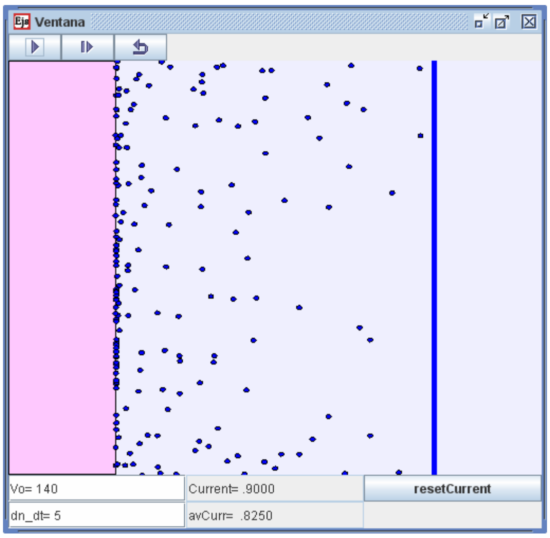

Figure 4: Interface of the simulation of a diode, composed of two planar parallel electrodes. Emitting cathode at left and anode at right. The spatial charge is shown.

For a given temperature of the cathode, a saturation of the current occurs as the applied potential increases (Hemenway 1962). In our study we can simulate this fact modifying the rate of particles injected into the space. In figure 5 the current versus the applied voltage for different temperatures are shown, these curves had been obtained with the simulation. The saturation of the current is clearly appreciated and the variation with the temperature agrees qualitatively with the experiments.

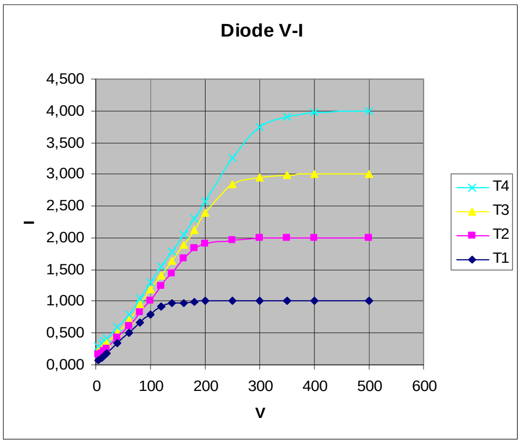

Figure 5: Current versus the applied voltage for different temperatures. Data obtained from the simulation

The relationship between the electric current and the voltage is well known and was established by Child and Langmuir (Owen 1929) in the three-half power law for the non saturated region.

$$J = 2.33 \times 10^{-6}\ V^{3/2}/d^2\ A/m^2 \qquad (4)$$

Where *d* is the distance from the cathode to the anode. The curves we obtain are qualitatively coincident with Child Langmuir law, but with quantitative deviations in respect to the 3/2 law. This is so because we have not been able of properly modeling the cathode and anode metallic electrodes (as constant potential bodies in the presence of the emitted electrons). Work in this respect is being carried out.

5. Conclusions

The thermionic emission in 2D and the characteristic curve of the diode are usually studied using statistical physics and advanced electromagnetic partial differential equations, we have studied these subjects with a simulation that use a model in terms of some few well known basic laws. We use an ideal gas to study the Richardson law, the dependence of current density with work function looks nice, considering Boltzmann distribution instead of the Fermi one drives to a dependence of current density with the square of temperature instead of power two. A model of charged particles interacting between them and with an external electric field drives to an acceptable dependence of the current with the voltage, some improvement must be done to model the electrodes.
Active learning is a must in the teaching process, these simulations are very useful to push students to put their "hands" on these subjects. The teacher could guide the student in his research reproducing the steps described in the previous paragraphs.
Applets of the simulations can be downloaded from:
http://webs.um.es/jmz/jmz/thermionic/index.html, once you have the applet you can get its source code using Ejs (Ejs 2009).

References

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