STUDY OF STABILITY MATTER PROBLEM IN MICROPOLAR GENERALISED THERMOELASTIC

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Abstract

The theory of micropolar thermoelasticity has many applications. One form of the recent years concerning the problem of propagation of thermal waves at finite speed and the possibility of “second sound” effects established a new thermo mechanical theory of deformable media that uses a general entropy balance as postulated and the
theory is illustrated in detail in the context of flow of heat in a rigid solid, with particular reference to the propagation of thermal waves at finite speed. Then theory of thermoelasticity for non-polar bodies, based on the new procedures, was discussed and employed the eigen value approach to study the effect of rotation and relaxation time in two dimensional problem of generalized thermoelasticity. Recently investigation shows the dynamic response of a homogeneous, isotropic, generalized thermoelastic half-space with voids subjected to normal, tangential force and thermal stress. In this paper we introduce the eigen value approach, following Laplace and Fourier transformation has been employed to find the general solution of the field equation in a micropolar generalized thermoelastic medium for plane strain problem. An application of an infinite space with an impulsive mechanical source has been taken to illustrate the utility of the approach. The integral transformation has been inverted by using a numerical inversion technique to get result in physical domain. The result in the form of normal displacement, normal force stress, tangential force stress, tangential couple stress and temperature field components have been obtained numerically and illustrated graphically. Special case of a thermoelastic solid has also been deduced.

Keywords: Eigen value; Mechanical sources ; Laplace and Fourier transform ; Concentrated force; Romberg’s integration

1. Introduction

The theory of micropolarthermoelasticity has been a subject of intensive study. A comprehensive review of works on the subject was given by Eringen (1970) and Nowacki (1966). There has been very much written in recent years concerning the problem of propagation of thermal waves at finite speed. A generalized theory of linear micropolarthermoelasticity that admits the possibility of “second sound” effects was established by Boschi (1993). Recently, Green and Naghdi (1991) established a new thermomechanical theory of deformable media that uses a general entropy balance as postulated by Green and Naghdi (1977). The theory is illustrated in detail in the context of flow of heat in a rigid solid, with particular reference to the propagation of thermal waves at finite speed. A theory of thermoelasticity for non-polar bodies, based on the new procedures, was discussed by Green and Naghdi (1993) .Bakshi, Bera and Debnath (2004) employed the eigen value approach to study the effect of rotation and relaxation time in two dimensional problem of generalized thermoelasticity. Kumar and Rani (2004) studied the deformation due to mechanical and thermal sources in generalized orthorhomticthermoelastic material. Kumar and Rani (2005) investigated the dynamic response of a homogeneous, isotropic, generalized thermoelastic half-space with voids subjected to normal, tangential force and thermal stress. The micropolar theory was extended to include thermal effects by Nowacki (1966) and Eringen (1970).

Kumar and Chadha (1986) derived the expressions for displacements, microrotation, force stress, couple stress and first moment for a half - space subjected to an arbitrary temperature field and a particular case of line heat source has been discussed in detail. The uniqueness of the solution of some boundary value problems of the linear micropolarthermoelasticity was investigated by Cracium (1990). Passarella (1996) solved the initial-boundary value problem for micropolarthermoelasticity and proved a uniqueness theorem for the problem. Mahalanabis and Manna (1997) discussed eigen value approach to linear micropolarthermoelasticity by arranging basic equations of elasticity in the form of matrix deferential equation in the Hankel transform and extended the approach to linear thermoelasticity. Marin and Lupu (1998) investigated harmonic vibrations in thermoelasticity of micropolar bodies. Kumar and Deswal (2001) discussed the disturbance due to mechanical and thermal sources in homogeneous isotropic micropolar generalized thermoelastic half-space.

2. Formulation and solution of the Problem

We consider a homogeneous, isotropic, micropolar generalized thermoelastic solid in an undisturbed state and initially at uniform temperature. We take a cartesian system (x,y,z) and z-axis pointing vertically into the medium.

Following Eringen (1968), Lord and Shulman (1967) and Green and Lindsay (1972), the field equations and the constitutive relations in micropolar generalized thermoelastic solid without body forces, body couples and heat sources can be written as

$$(\lambda+2\mu+K)\nabla(\nabla.\boldsymbol{u})-(\mu+K)\nabla\times\nabla\times\boldsymbol{u}+K\nabla\times\boldsymbol{\phi}-\mathrm{v}\left(1+\tau_1\frac{\partial}{\partial t}\right)\nabla\mathrm{T}=\rho\frac{\partial^2\mathbf{u}}{\partial t^2}$$

$$(\alpha+\beta+\gamma)\nabla(\nabla.\boldsymbol{\phi})-\gamma\nabla\times\nabla\times\boldsymbol{\phi}+K\nabla\times\mathbf{u}-2\mathrm{K}\boldsymbol{\phi}=\rho\mathrm{j}\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}$$

$$K^*\nabla^2T=\rho C^*\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2\mathrm{T}}{\partial t^2}\right)+\mathrm{v}T_0\left(\frac{\partial}{\partial t}+\Xi\,\tau_0\frac{\partial^2}{\partial t^2}\right)\nabla\cdot\mathbf{u}$$

$$m_{ij}=\alpha\phi_{r,r}\delta_{ij}+\beta\,\phi_{i,j}+\gamma\phi_{j,i}\ ,$$

$$t_{ij}=\lambda u_{r,r}\delta_{ij}+\mu\left(u_{i,j}+u_{j,i}\right)+K\left(u_{j,i}-\varepsilon_{ijr}\phi_r\right)-\nu\left(\mathrm{T}+\tau_1\frac{\partial T}{\partial t}\right)\delta_{ij}\ ,$$

For the L-S (Lord Shulman) theory $\tau_1=0$, $\Xi=1$ and for G - L (Green Lindsay) theory $\tau_1=0$, $\Xi=0$,

The thermal relaxations τ_0 and τ_1 satisfy the inequality $\tau_1\geq\tau_0>0$ for the G-L theory only. However, it has been proved by Sturnin (2001) that the inequalities are not mandatory for τ_0 and τ_1 to follow.

For two dimensional plane strain problem parallel to*xz*-plane, we assume

$$\boldsymbol{u}=(u_1,0,u_3),\qquad \boldsymbol{\phi}=(0,\phi_2,0)$$

The displacement components u_1, u_3 and microrotation component ϕ_2 depend upon x, z and t and are independent of co-ordinate y, so that $\frac{\partial}{\partial y} \equiv 0$. With these considerations and using (2.6) and introducing the non-dimensional quantities as

$$x' = \frac{\omega^* x}{C_1}, \qquad z' = \frac{\omega^* z}{C_1},$$

$$T' = \frac{T}{T_0}, \qquad u_1' = \frac{\rho \omega^* C_1 u_1}{\nu T_0},$$

$$m_{32}' = \frac{\omega^*}{C_1 \nu T_0} m_{32},$$

Where

$$\omega^* = \frac{C^* (\lambda + 2\mu)}{K^*}, \qquad C_1^2 = \frac{\lambda + 2\mu}{\rho}.$$

Now applying Laplace and Fourier transform defined by

$$\overline{f}(x,z,p) = \int_0^\infty f(x,z,t) \exp(-pt) dt,$$

$$\tilde{f}(\xi,z,p) = \int_{-\infty}^\infty \overline{f}(x,z,p) \, \mathrm{e}(-\iota \xi x) dx,$$

on the set of equations (2.1)-(2.3), after suppressing primes, we get

$$\frac{d^2 \tilde{u}_1}{dz^2} = \frac{1}{\mathrm{m}_3}\left[\left(\mathrm{m}_1 \xi^2 + \mathrm{p}^2\right)\tilde{u}_1 - \iota \xi \mathrm{m}_2 \frac{d\tilde{u}_3}{dz} + \mathrm{m}_4 \frac{d\tilde{\phi}_2}{dz} + \iota \xi (1 + \tau_1 p) \tilde{\mathrm{T}} \right]$$

$$\frac{d^2 \tilde{u}_3}{dz^2} = \frac{1}{\mathrm{m}_1}\left[-\iota \ \mathrm{m}_2 \xi \frac{d\tilde{u}_1}{dz} + \left(\mathrm{m}_3 \xi^2 + \mathrm{p}^2\right)\tilde{u}_3 - \mathrm{m}_4 \iota \xi \tilde{\phi}_2 + \frac{d\tilde{\mathrm{T}}_1}{dz} \right]$$

$$\frac{d^2 \tilde{\phi}_2}{dz^2} = -\mathrm{m}_5 \frac{d\tilde{u}_1}{dz} + \iota \xi \mathrm{m}_5 \tilde{u}_3 + \left(2\mathrm{m}_5 + \xi^2 + \mathrm{m}_6 \mathrm{p}^2\right)\tilde{\phi}_2$$

$$\frac{d^2\tilde{T}}{dz^2} = \varepsilon\ p(1+\tau_0 p\Xi)\left\{\iota\,\xi\,\tilde{u}_1 + \frac{d\,\tilde{u}_3}{dz}\right\} + \left\{\xi^2 + p(1+\tau_0 p)\tilde{T}\ \right\}$$

Where

$$m_1 = \frac{\lambda + 2\mu + K}{\rho\, C_1^2}\ , \qquad m_2 = \frac{\lambda + \mu}{\rho\ C_1^2},$$

$$m_4 = \frac{K}{\rho\ C_1^2}, \qquad m_5 = \frac{KC_1^2}{\rho\omega^{*2}},$$

$$m_7 = \frac{\mu}{\rho\ C_1^2}, \qquad m_8 = \frac{\lambda}{\rho\ C_1^2},$$

$$\varepsilon = \frac{\mathrm{T}_0\ \beta_1^2}{\rho\ K^*\omega^*}.$$

Equations (2.9) - (2.12) can be written in the vector matrix differential equation form as

$$\frac{d}{dz}W(\xi, z, p) = A(\xi, p)W(\xi, z, p)$$

Where

$$W = \begin{bmatrix} U \\ DU \end{bmatrix}, \qquad A = \begin{bmatrix} O & I \\ A_2 & A_1 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} 0 & f_{12} & f_{13} & 0 \\ f_{21} & 0 & 0 & f_{24} \\ f_{31} & 0 & 0 & 0 \\ 0 & f_{42} & 0 & 0 \end{bmatrix}, \qquad A_2 = \begin{bmatrix} g_{11} & 0 & 0 & g_{14} \\ 0 & g_{22} & g_{23} & 0 \\ 0 & g_{32} & g_{33} & 0 \\ g_{41} & 0 & 0 & g_{44} \end{bmatrix}$$

and O is the Null matrix of order 4 with

$$f_{12} = \frac{-l\xi\ m_2}{m_3}, \qquad f_{13} = \frac{m_4}{m_3},$$

$$f_{31} = -m_5, \qquad f_{42} = \varepsilon p(1+\tau_0 p\ \Xi),$$

$g_{22} = \frac{\left(m_3\xi^2 + p^2\right)}{m_1}$, $g_{22} = \frac{-l\, m_4\ \xi}{m_1}$,

$g_{41} = l\varepsilon\ \xi\ p\left(1+\tau_0 p\Xi\right)$, $g_{44} = \ \xi^2 + \ p\left(1+\tau_0 p\right)$,

To solve the equation (2.14), we take

$W(\xi, z, p) = X(\xi, p)\ e^{qz}$ for some q, So we obtain

$A(\xi\ , p)W(\xi\ , z, p) = q\, W(\xi\ , z, p)$,

This leads to Eigen value problem. The characteristic equation corresponding to the matrix A is given by

$$det(A - q\mathrm{I}) = 0$$

which on expansion provides us

$q^8 - \sigma_1 q^6 + \sigma_2 q^4 - \sigma_3 q^2 + \sigma_4 = 0$

Where

$\sigma_1 = g_{11} + g_{22} + g_{33} + g_{44} + f_{24} f_{42} + f_{12} f_{21} + f_{13} f_{31}$,

$$\sigma_2 = g_{11}g_{22} + g_{22}g_{33} + g_{33}g_{11} + g_{44}g_{11} + g_{44}g_{22} + g_{44}g_{33} + f_{12} f_{24} g_{41} + f_{24} f_{42} g_{11} + f_{24} f_{42} g_{33} + f_{24} f_{42} f_{13} f_{31} - g_{32}g_{23} + f_{12} f_{21} g_{33} + f_{12} f_{21} g_{44} - f_{12} f_{31} g_{23} + f_{13} f_{31} g_{22} + f_{13} f_{31} g_{44} - g_{14}g_{41} - g_{14} f_{21} f_{42},$$

$$\sigma_3 = g_{11}g_{22}g_{33} + g_{22}g_{33}g_{44} + g_{33}g_{44}g_{11} + g_{44}g_{11}g_{22} - g_{11}g_{23}g_{32} - g_{44}g_{23}g_{32} + f_{24} f_{42} g_{11} g_{33} + f_{13} f_{24} g_{41} g_{32} + f_{31} f_{42} g_{14} g_{23} + f_{12} f_{21} g_{33} g_{44} - f_{13} f_{21} g_{32} g_{44} - f_{12} f_{31} g_{23} g_{44} - f_{42} f_{21} g_{14} g_{33} + f_{13} f_{31} g_{22} g_{44} - g_{14} g_{41} g_{22} - g_{14} g_{41} g_{33},$$

$$\sigma_4 = g_{11}g_{22}g_{33}g_{44} - g_{23}g_{32}g_{11}g_{44} + g_{14}g_{41}g_{22}g_{33} + g_{23}g_{32}g_{14}g_{41}.$$

The eigen values of the matrix A are the characteristic roots of the equation (2.19). The vectors $\mathrm{X}\left(\xi, p\right)$ corresponding to the eigen values q_s can be determined by solving the homogeneous equations

$\left[A - qI\right] X_s\left(\xi, p\right) = 0$

The set of eigen vectors $\mathrm{X_s}\left(\xi,\mathrm{p}\right)$; s =1, 2, 3, ..., 8 may be defined as

$$X_s(\xi, p) = \begin{bmatrix} X_{s1}(\xi, p) \\ X_{s2}(\xi, p) \end{bmatrix}$$

Where

$$X_{s1}(\xi, p) = \begin{bmatrix} a_s q_s \\ b_s \\ -\xi \\ c_s \end{bmatrix}, \qquad X_{s2}(\xi, p) = \begin{bmatrix} a_s q_s^2 \\ b_s q_s \\ -\xi\, q_s \\ c_s q_s \end{bmatrix},$$

$$X_{l1}(\xi, p) = \begin{bmatrix} -a_s q_s \\ b_s \\ -\xi \\ c_s \end{bmatrix}, \qquad X_{l2}(\xi, p) = \begin{bmatrix} a_s q_s^2 \\ -b_s q_s \\ \xi\, q_s \\ -c_s q_s \end{bmatrix},$$

$$a_s = \frac{-\xi}{m_3 \Delta_s}[\{\xi^2 + p(1+\tau_0 p) - q_s^2\}\{m_4 m_5 + m_2(2m_5 + \xi^2 + p^2 m_6 - q_s^2) + \varepsilon\, p(2m_5 + \xi^2 + p^2 m_6 - q_s^2)(1+\tau_1 p)(1+\tau_0 p)\Xi\}]$$

$$b_s = \frac{\iota}{m_3 \Delta_s}[\{\xi^2 + p(1+\tau_0 p) - q_s^2\}\{m_4 m_5 q_s^2 + (2m_5 + \xi^2 + p^2 m_6 - q_s^2)(m_1 \xi^2 + p^2 - m_3 q_s^2)\} + \varepsilon\, p\, \xi^2 (2m_5 + \xi^2 + p^2 m_6 - q_s^2)(1+\tau_1 p)(1+\tau_0 p)\Xi]$$

$$c_s = \frac{\varepsilon p q_s (1+\tau_0 p\Xi)(\iota \xi a_s + b_s)}{[q_s^2 - \{\xi^2 + p(1+\tau_0 p)\}]}$$

$$\Delta_s = \frac{m_5}{m_3}[\{\xi^2 + p(1+\tau_0 p) - q_s^2\}\{m_2 q_s^2 - (m_1 \xi^2 + p^2 - q_s^2 m_3)\} + \varepsilon\, p(1+\tau_0 p\Xi)(1+\tau_1 p)(q_s^2 - \xi^2)]$$

Thus solution of equation (2.14) is as given by Sharma and Chand.(1992)

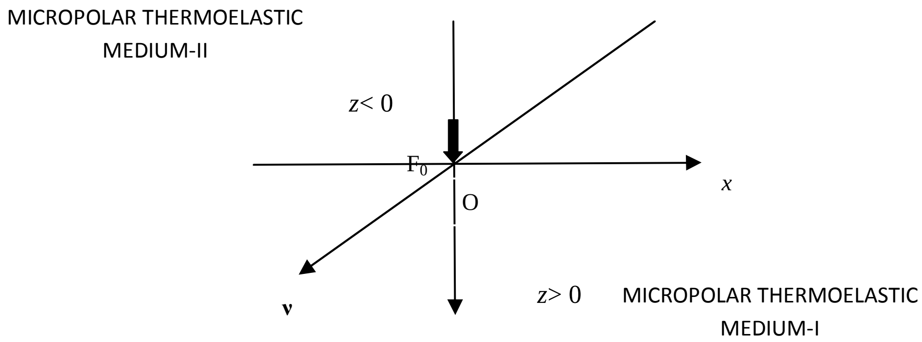
$$W(\xi, z, p) = \sum_{s=1}^{4}\left[E_s X_s(\xi, p) e^{q_s z} + E_{s+4} X_{s+4}(\xi, p) e^{-q_s z}\right]$$

$E_1, E_2, E_3, E_4, E_5, E_6, E_7$ and E_8 are eight arbitrary constants. The equation (2.32) represents a general solution of the plane strain problem for isotropic, micropolar generalized thermoelastic solid and gives the displacement, microrotation and temperature field in the transformed domain.

3.Applications

Mechanical Source

We consider an infinite micropolar generalized thermoelastic space in which a concentrated force where F_0 is the magnitude of the force, $F_z = -F_0\delta(x)\delta(t)$ acting in the direction of the z-axis at the origin of the Cartesian co-ordinate system as shown in fig.1. The boundary condition for present problem on the plane z=0 are



$$u_1(x, 0^+, t) - u_1(x, 0^-, t) = 0\ ,\quad u_3(x, 0^+, t) - u_3(x, 0^-, t) = 0,$$

$$\phi_2(x, 0^+, t) - \phi_2(x, 0^-, t) = 0\ ,\quad T(x, 0^+, t) - T(x, 0^-, t) = 0,$$

$$\frac{\partial T}{\partial z}\left(x, 0^+, t\right) - \frac{\partial T}{\partial z}\left(x, 0^-, t\right) = 0\ ,\quad t_{31}\left(x, 0^+, t\right) - t_{31}\left(x, 0^-, t\right) = 0,$$

$$t_{33}\left(x, 0^+, t\right) - t_{33}\left(x, 0^-, t\right) = -F_0\delta(x)\delta(t)\ ,\quad m_{32}\left(x, 0^+, t\right) - m_{32}\left(x, 0^-, t\right) = 0$$

Making use of equations (2.6)-(2.7) and $F_0' = \frac{F_0}{K}$ in equations (2.4)-(2.5), we get the stresses in the non-dimensional form with primes. After suppressing the primes, we apply Laplace and Fourier transforms defined by equations (2.8) on the resulting equations and from equation (3.1), we get transformed components of displacement, microrotation, temperature field, tangential force stress, normal force stress and tangential couple stress for z > 0 are given by

$$\tilde{u}_1(\xi, z, p) = -\left\{a_1 q_1 E_5 e^{-q_1 z} + a_2 q_2 E_6 e^{-q_2 z} + a_3 q_3 E_7 e^{-q_3 z} + a_4 q_4 E_8 e^{-q_4 z}\right\},$$

$$\tilde{u}_3(\xi, z, p) = b_1 E_5 e^{-q_1 z} + b_2 E_6 e^{-q_2 z} + b_3 E_7 e^{-q_3 z} + b_4 E_8 e^{-q_4 z},$$

$$\tilde{\varphi}_2(\xi, z, p) = -\xi\left\{E_5 e^{-q_1 z} + E_6 e^{-q_2 z} + E_7 e^{-q_3 z} + E_8 e^{-q_4 z}\right\},$$

$$\tilde{T}(\xi, z, p) = c_1 E_5 e^{-q_1 z} + c_2 E_6 e^{-q_2 z} + c_3 E_7 e^{-q_3 z} + c_4 E_8 e^{-q_4 z},$$

$$\tilde{t}_{31}(\xi, z, p) = \left(m_3 a_1 q_1^{\,2} + \iota\xi\, b_1 s_{10} + \xi\, m_4\right) E_5 e^{-q_1 z} + \left(m_3 a_2 q_2^{\,2} + \iota\xi\, b_2 m_7 + \xi\, m_4\right) E_6 e^{-q_2 z} + \left(m_3 a_3 q_3^{\,2} + \iota\xi\, b_3 m_7 + \xi\, m_4\right) E_7 e^{-q_3 z} + \left(m_3 a_4 q_4^{\,2} + \iota\xi\, b_4 m_7 + \xi\, m_7\right) E_8 e^{-q_4 z},$$

$$\begin{aligned}\tilde{t}_{33}(\xi, z, p) = -[&\left(\iota\xi m_8 a_1 q_1 + m_1 b_1 q_1 + c_1\left(1+\tau_1 p\right)\right) E_5 e^{-q_1 z} \\ &+\left(\iota\xi\, m_8 a_2\, q_2 + m_1 b_2 q_2 + c_2\left(1+\tau_1 p\right)\right) E_6 e^{-q_2 z} \\ &+ \left(\iota\xi\, m_8 a_3 q_3 + m_1 b_3 q_3 + c_3\left(1+\tau_1 p\right)\right) E_7 e^{-q_3 z} \\ &+\left(\iota\xi\, m_8 a_4 q_4 + m_1 b_4 q_4 + c_4\left(1+\tau_1 p\right)\right) E_8 e^{-q_4 z},\end{aligned}$$

$$\tilde{m}_{32}(\xi, z, p) = \xi\, s_8 \left\{q_1 E_5 e^{-q_1 z} + q_2 E_6 e^{-q_2 z} + q_3 E_7 e^{-q_3 z} + q_4 E_8 e^{-q_4 z}\right\},$$

for z<0, the above expressions get suitably modified, e.g.

$$\tilde{u}_1(\xi, z, p) = a_1 q_1 E_1 e^{q_1 z} + a_2 q_2 E_2 e^{q_2 z} + a_3 q_3 E_3 e^{q_3 z} + a_4 q_4 E_4 e^{q_4 z},$$

Making use of the transformed displacements, microrotation, microstretch and stresses given by (3.6)-(3.12) in the transformed boundary conditions, we obtain eight linear relations between the E_i's, which on solving gives

$$E_1 = E_5 = \frac{F_0}{2q_1\Delta_1}\left[c_2\left(a_3 - a_4\right) + c_3\left(a_4 - a_2\right) + c_4\left(a_2 - a_3\right)\right],$$

$$E_2 = E_6 = \frac{F_0}{2q_2\Delta_1}\left[c_1\left(a_4 - a_3\right) + c_3\left(a_1 - a_4\right) + c_4\left(a_3 - a_1\right)\right],$$

$$E_3 = E_7 = \frac{F_0}{2q_3\Delta_1}\left[c_1\left(a_2 - a_4\right) + c_2\left(a_4 - a_1\right) + c_4\left(a_1 - a_2\right)\right],$$

$$E_4 = E_8 = \frac{F_0}{2q_4\Delta_1}\left[c_1\left(a_3 - a_2\right) + c_2\left(a_1 - a_3\right) + c_3\left(a_2 - a_1\right)\right],$$

Where

$$\begin{aligned}\Delta_1 = m_1[&c_1\left\{\left(a_2 b_3 - a_3 b_2\right) + \left(a_3 b_4 - a_4 b_3\right) + \left(a_4 b_2 - a_2 b_4\right)\right\} \\ &+ c_2\left\{\left(a_3 b_1 - a_1 b_3\right) + \left(a_1 b_4 - a_4 b_1\right) + \left(a_4 b_3 - a_3 b_4\right)\right\} \\ &+ c_3\left\{\left(a_1 b_2 - a_2 b_1\right) + \left(a_4 b_1 - a_1 b_4\right) + \left(a_2 b_4 - a_4 b_2\right)\right\} \\ &+ c_4\left\{\left(a_2 b_1 - a_1 b_2\right) + \left(a_1 b_3 - a_3 b_1\right) + \left(a_3 b_2 - a_2 b_3\right)\right\}],\end{aligned}$$

Thus functions $\tilde{u}_1, \tilde{u}_3, \tilde{\phi}_2, \tilde{T}, \tilde{t}_{31}, \tilde{t}_{33}$ and $\tilde{m}_{32}$ have been determined in the transformed domain and these enable us to find the displacements, microrotation, temperature field and stresses.

Case I : For L-S theory, a_s , b_s and c_s in the expressions (3.5)-(3.12) take the form

$$a_s = \frac{-\xi}{\mathrm{m_3\Delta_s}} \; [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^2\right\}\{m_4 m_5 + m_2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right) +$$
$$\varepsilon\, p\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(1+\tau_0 p\right)\}\;],$$

$$b_s = \frac{\iota}{\mathrm{m_3\Delta_s}} \; [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^2\right\}\{m_4 m_5 q_s^2 + \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(m_1\xi^2 + p^2 - m_3 q_s^2\right)\} +$$
$$\varepsilon\; p\;\xi^2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(1+\tau_0 p\right)],$$

$$c_s = \frac{\varepsilon p q_s\left(\iota\xi a_s + b_s\right)}{\left[q_s^2 - \left\{\xi^2 + p\;\left(1+\tau_0 p\right)\right\}\right]},$$

Where

$$\Delta_s = \frac{\mathrm{m_5}}{\mathrm{m_3}} \; [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^2\right\}\{m_2 q_s^2 - \left(m_1\xi^2 + p^2 - q_s^2 m_3\right)\;\} +$$
$$\varepsilon\; p\;\left(1+\tau_1 p\right)\left(q_s^2 - \xi^2\right)\;]\;; \qquad \mathrm{s}=1,2,3,4$$

and $\pm q_s$ (s =1, 2, 3, 4) are roots of the equation (2.19) in which $\sigma_1, \sigma_2, \sigma_3$ *and* σ_4 are obtained respectively from expressions (2.20)-(2.23) by taking $\tau_1 = 0$, $\Xi = 1$.

Case II : For G-L theory, as b's and c's in the expressions (3.5)-(3.12) take the form

$$a_s = \frac{-\xi}{\mathrm{m_3\Delta_s}} \; [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^2\right\}\{m_4 m_5 + m_2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right) +$$
$$\varepsilon\, p\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(1+\tau_1 p\right)\}\;],$$

$$b_s = \frac{\iota}{\mathrm{m_3\Delta_s}} \; [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^2\right\}\{m_4 m_5 q_s^2 + \left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(m_1\xi^2 + p^2 - m_3 q_s^2\right)\;\} +$$
$$\varepsilon\; p\;\xi^2\left(2m_5 + \xi^2 + p^2 m_6 - q_s^2\right)\left(1+\tau_1 p\right)\;],$$

$$c_s = \frac{\varepsilon p q_s\left(\iota\xi a_s + b_s\right)}{\left[q_s^2 - \left\{\xi^2 + p\;\left(1+\tau_0 p\right)\right\}\right]},$$

Where

$$\Delta_s = \frac{m_5}{m_3}\ [\{\xi^2 + p(1+\tau_0 p) - q_s^2\}\{m_2 q_s^2 - (m_1\xi^2 + p^2 - q_s^2 m_3)\ \} +$$

$$\varepsilon\ p\ (1+\tau_1 p)(q_s^2 - \xi^2)\ ]\ ; \qquad s = 1,2,3,4$$

and $\pm q_s$ (s =1, 2, 3, 4) are roots of the equation (2.19) in which $\sigma_1, \sigma_2, \sigma_3$ *and* σ_4 are obtained respectively from expressions (2.20) - (2.23) by taking $\Xi = 0$

Case III : For Green and Naghdi theory (G-N), equations (2.1), (2.3) and (2.4) can be written as

$$(\lambda + 2\mu + K)\nabla(\nabla.\mathbf{u}) - (\mu + K)\nabla \times \nabla \times \boldsymbol{u} + K\nabla \times \boldsymbol{\phi} - \nu\nabla \mathrm{T} = \rho \frac{\partial^2 \mathbf{u}}{\partial \mathrm{t}^2}$$

$$\mathrm{K}^*\nabla^2\mathrm{T} = \rho \mathrm{C}^* \frac{\partial^2 \mathrm{T}}{\partial \mathrm{t}^2} + \nu T_0 \frac{\partial^2(\nabla.\mathbf{u})}{\partial \mathrm{t}^2}$$

$$t_{ij} = \lambda\, u_{r,r}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\phi_r) - \nu T\delta_{ij}$$

and K^* is not the usual thermal conductivity but a material characteristics constant in G - N & theory and is given $K^*\left(= \frac{C^*(\lambda + 2\mu)}{4} \right)$

With the help of equations (3.26)-(3.28) and following the procedure of the previous sections, we get the expressions for displacements, microrotation, temperature, field, force stresses and couple stress by taking in equations (3.5)-(3.11).

$$(1+\tau_1 p) = 1, \qquad\qquad (1+\tau_0 p) = 4p,$$

Particular Case I : Neglecting micropolarity effect i.e. $\alpha = \beta = \gamma = K = j = 0$ in equations (3.5)-(3.12), the expressions for displacement components, force stresses and temperature field are obtained in a thermoelastic medium as

$$\tilde{\mathrm{u}}_1(\xi, \mathrm{z}, \mathrm{p}) = -\{\mathrm{a}_1^*\mathrm{q}_1\mathrm{E}_4^* e^{-\mathrm{q}_1^* z} + \mathrm{a}_2^*\mathrm{q}_2^*\mathrm{E}_5^* e^{-\mathrm{q}_2^* z} + \mathrm{a}_3^*\mathrm{q}_3^*\mathrm{E}_6^* e^{-\mathrm{q}_3^* z}\},$$

$$\tilde{\mathrm{u}}_3(\xi, \mathrm{z}, \mathrm{p}) = \mathrm{b}_1^*\mathrm{E}_4^* e^{-\mathrm{q}_1^* z} + \mathrm{b}_2^*\mathrm{E}_5^* e^{-\mathrm{q}_2^* z} + \mathrm{b}_3^*\mathrm{E}_6^* e^{-\mathrm{q}_3^* z},$$

$$\tilde{T}(\xi, \mathrm{z}, \mathrm{p}) = -\xi\ \{\mathrm{E}_4^* e^{-\mathrm{q}_1^* z} + \mathrm{E}_5^* e^{-\mathrm{q}_2^* z} + \mathrm{c}_3\mathrm{E}_6^* e^{-\mathrm{q}_3^* z}\},$$

$$\tilde{\mathrm{t}}_{31}(\xi, \mathrm{z}, \mathrm{p}) = \mathrm{m}_7\{\ (\mathrm{a}_1^*\mathrm{q}_1^{*2} + \iota\xi\ \mathrm{b}_1^*)\ \mathrm{E}_4^* e^{-\mathrm{q}_1^* z} + (\mathrm{a}_2^*\mathrm{q}_2^{*2} + \iota\xi\ \mathrm{b}_2^*)\ \mathrm{E}_5^* e^{-\mathrm{q}_2^* z} + (\mathrm{a}_3^*\mathrm{q}_3^{*2} + \iota\xi\ \mathrm{b}_3^*)\ \mathrm{E}_6^* e^{-\mathrm{q}_3^* z}\},$$

$$\tilde{t}_{33}(\xi, z, p) = -[\left\{m_1^* q_1^* b_1^* + \iota\xi m_8 a_1^* q_1^* - \xi\left(1+\tau_1 p\right)\right\} E_4^* e^{-q_1^* z} +$$

$$\left\{m_1^* q_2^* b_2^* + \iota\xi m_8 a_2^* q_2^* - \xi\left(1+\tau_1 p\right)\right\} E_5^* e^{-q_2^* z} +$$

$$\left\{m_1^* q_3^* b_3^* + \iota\xi m_8 a_3^* q_3^* - \xi\left(1+\tau_1 p\right)\right\} E_6^* e^{-q_3^* z}],$$

Where

$$E_1^* = E_4^* = \frac{F_0\left(a_3^* - a_2^*\right)}{2q_1^* \Delta_1^*},$$

$$E_2^* = E_5^* = \frac{F_0\left(a_1^* - a_3^*\right)}{2q_2^* \Delta_1^*},$$

$$E_3^* = E_6^* = \frac{F_0\left(a_2^* - a_1^*\right)}{2q_3^* \Delta_1^*},$$

$$\Delta_1^* = m_1^* \left\{\left(a_2^* b_3^* - a_3^* b_2^*\right) + \left(a_3^* b_1^* - a_1^* b_3^*\right) + \left(a_1^* b_2 - a_2^* b_1^*\right)\right\},$$

$$a_s^* = \frac{-\xi}{m_3^* \Delta_s^*}\ [m_2\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^{*2}\right\} + \ \varepsilon\ p\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right)\}],$$

$$b_s^* = \frac{\iota}{m_3^* \Delta_s^*}\ [\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^{*2}\right\}\left(m_1^* \xi^2 + p^2 - m_3^* q_s^{*2}\right)\ \} + \ \varepsilon\ p\ \ \xi^2\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right)],$$

$$\Delta_s^* = \frac{\varepsilon\ p q_s^{*2}\ \left(1+\tau_0 p\Xi\right)}{\xi\ m_3^*}\ \left\{\left(m_1^* \xi^2 + p^2 - q_s^{*2} m_3^*\right) - \xi^2 m_2\right\},$$

and $\pm q_s^*\left(s = 1, 2, 3\right)$ are the roots of the equation

$$q^{*6} - \sigma_1^* q^{04} + \sigma_2^* q^{*2} - \sigma_3^* = 0$$

$$\sigma_1^* = \left(\frac{m_1^* \xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^* \xi^2 + p^2}{m_1^*}\right)\ + \left\{\xi^2 + p\left(1+\tau_0 p\right)\right\} + \frac{\varepsilon p}{m_1^*}\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right) - \frac{\xi^2 m_2^2}{m_1^* m_3^*},$$

$$\sigma_2^* = \left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) + \left(\frac{m_3^*\xi^2 + p^2}{m_1^*}\right) + \left(\frac{m_3^*\xi^2 + p^2}{m_1^*}\right)\left\{\xi^2 + p\left(1+\tau_0 p\right)\right\} +$$
$$\left\{\xi^2 + p\left(1+\tau_0 p\right)\right\}\left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) + \frac{\varepsilon p}{m_1^*}\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right)\left(\frac{m_1^*\xi^2 + p^2}{m_3^*}\right) -$$
$$\frac{\xi^2 m_2^2}{m_1^* m_3^*}\left\{\xi^2 + p\left(1+\tau_0 p\right)\right\} - \frac{2\varepsilon p \xi^2 m_2}{m_1^* m_3^*}\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right) + \frac{\varepsilon p \xi^2}{m_3^*}\left(1+\tau_0 p\Xi\right)\left(1+\tau_1 p\right),$$

With

$$m_1^* = \frac{\lambda + 2\mu}{\rho C_1^2}, \qquad m_3^* = \frac{\mu}{\rho C_1^2}$$

i. For L-S theory : Taking $\tau_1 = 0$, $\Xi = 1$in expression given by (3.29)-(3.33) of particular case I, we obtain expressions for displacement components, temperature field and force stresses.

ii. For G-L theory : Taking $\Xi = 0$ in expressions given by (3.29)-(3.33)of particular case I, we obtain expressions for displacement components, temperature field and force stresses

iii. For G-N theory : Neglecting micropolarity effect i.e. ($\alpha = \beta = \gamma = K = j = 0$) in subcase III of case I, we get the expressions for displacement components, temperature field and force stresses are obtained in a thermoelastic medium by taking

$$1+\tau_1 p = 1, \qquad 1+\tau_0 p = 4p, \qquad 1+\tau_0 p\Xi = p, \quad \varepsilon = \frac{\varepsilon_1}{4}, \quad \omega^* = \frac{C_1}{h}, \quad \varepsilon_1 = \frac{T_0\ \nu^2}{\rho\ K^*}$$

in equations (3.29)-(3.44) as

$$\tilde{u}_1(\xi, z, p) = -\left\{a_1^0 q_1^0 E_4^0 e^{-q_1^0 z} + a_2^0 q_2^0 E_5^0 e^{-q_2^0 z} + a_3^0 q_3^0 E_6^0 e^{-q_3^0 z}\right\},$$

$$\tilde{u}_3(\xi, z, p) = b_1^0 E_4^0 e^{-q_1^0 z} + b_2^0 E_5^0 e^{-q_2^0 z} + b_3^0 E_6^0 e^{-q_3^0 z},$$

$$\tilde{T}(\xi, z, p) = -\xi\left\{E_4^0 e^{-q_1^0 z} + E_5^0 e^{-q_2^0 z} + c_3 E_6^0 e^{-q_3^0 z}\right\},$$

$$\tilde{t}_{31}(\xi, z, p) = m_7\{\left(a_1^0 q_1^{0^2} + \iota\xi\ b_1^0\right) E_4^0 e^{-q_1^0 z} + \left(a_2^0 q_2^{0^2} + \iota\xi\ b_2^0\right) E_5^0 e^{-q_2^0 z} + \left(a_3^0 q_3^{0^2} + \iota\xi\ b_3^0\right) E_6^0 e^{-q_3^0 z}\},$$

$$\tilde{t}_{33}(\xi, z, p) = -[\left\{m_1^0 q_1^0 b_1^0 + \iota\xi m_8 a_1^0 q_1^0 - \xi\left(1+\tau_1 p\right)\right\} E_4^0 e^{-q_1^0 z} + \left\{m_1^0 q_2^0 b_2^0 + \iota\xi m_8 a_2^0 q_2^0 - \xi\left(1+\tau_1 p\right)\right\} E_5^0 e^{-q_2^0 z}$$
$$+\left\{m_1^0 q_3^0 b_3^0 + \iota\xi m_8 a_3^0 q_3^0 - \xi\left(1+\tau_1 p\right)\right\} E_6^0 e^{-q_3^0 z}],$$

Where

$$E_1^0 = E_4^0 = \frac{F_0\left(a_3^0 - a_2^0\right)}{2q_1^0\Delta_1^0},$$

$$E_2^0 = E_5^0 = \frac{F_0\left(a_1^0 - a_3^0\right)}{2q_2^0\Delta_1^0},$$

$$E_3^0 = E_6^0 = \frac{F_0\left(a_2^0 - a_1^0\right)}{2q_3^0\Delta_1^0},$$

$$\Delta_1^0 = m_1^0\left\{\left(a_2^0 b_3^0 - a_3^0 b_2^0\right) + \left(a_3^0 b_1^0 - a_1^0 b_3^0\right) + \left(a_1^0 b_2 - a_2^0 b_1^0\right)\right\},$$

$$a_s^0 = \frac{-\xi}{\mathrm{m}_3^*\Delta_s^0}\ [\mathrm{m}_2\left\{\xi^2 + p\left(1+\tau_0 p\right) - q_s^{0^2}\right\} + \ \varepsilon_1\ p^2,$$

$$b_s^* = \frac{\iota}{\mathrm{m}_3^*\Delta_s^0}\ [\left\{\varepsilon_1 p^2\xi^2 + \left(\xi^2 + 4p^2 - q_s^{0^2}\right)\right\}\left(m_1^*\xi^2 + p^2 - m_3^* q_s^{0^2}\right)\}$$

$$\Delta_s^0 = \frac{\varepsilon_1\ p^2 q_s^0}{\xi\, m_3^*}\left\{\!\left(m_1^*\xi^2 + p^2 - m_3^* q_s^{0^2}\right) - \xi^2 m_2\right\}\ ,\ \varepsilon_1 = 4\varepsilon,$$

and $\pm q_s^0\left(s = 1,2,3\right)$ are the roots of the equation

$$q^{0^6} - \sigma_1^0 q^{0^4} + \sigma_2^0 q^{0^2} - \sigma_3^0 = 0,$$

$$\sigma_1^0 = \left(\frac{\mathrm{m}_1^*\xi^2 + p^2}{\mathrm{m}_3^*}\right) + \left(\frac{\mathrm{m}_3^*\xi^2 + p^2}{\mathrm{m}_1^*}\right)\ + \left(\xi^2 + 4p^2\right) + \frac{\varepsilon_1 p^2}{\mathrm{m}_1^*} - \frac{\xi^2\mathrm{m}_2^2}{\mathrm{m}_1^*\mathrm{m}_3^*},$$

$$\sigma_2^0 = \left(\frac{\mathrm{m}_1^*\xi^2 + p^2}{\mathrm{m}_3^*}\right) + \left(\frac{\mathrm{m}_3^*\xi^2 + p^2}{\mathrm{m}_1^*}\right)\ + \left(\frac{\mathrm{m}_3^*\xi^2 + p^2}{\mathrm{m}_1^*}\right)\left(\xi^2 + 4p^2\right) +$$
$$\left(\xi^2 + 4p^2\right)\left(\frac{\mathrm{m}_1^*\xi^2 + p^2}{\mathrm{m}_3^*}\right) + \frac{\varepsilon_1 p^2}{\mathrm{m}_1^*}\left(\frac{\mathrm{m}_1^*\xi^2 + p^2}{\mathrm{m}_3^*}\right) -$$
$$\frac{\xi^2\mathrm{m}_2^2}{\mathrm{m}_1^*\mathrm{m}_3^*}\left(\xi^2 + 4p^2\right) - \frac{2\varepsilon_1 p^2\xi^2\mathrm{m}_2}{\mathrm{m}_1^*\mathrm{m}_3^*} + \frac{\varepsilon_1 p^2\xi^2}{\mathrm{m}_3^*},$$

where $\tilde{f}_e$ and $\tilde{f}_0$ are even and odd parts of the functions $\tilde{f}(\xi, z, p)$ respectively.

Thus, expression (4.1) gives us the Laplace transform $\overline{f}(x, z, p)$ of the function $\mathrm{f}\,(\mathrm{x},\, \mathrm{z},\, \mathrm{t})$. Following Honig and Hirdes (1984), the Laplace transform function $\overline{f}(x, z, p)$ can be inverted to $\mathrm{f}\,(\mathrm{x},\, \mathrm{z},\, \mathrm{t})$.

$$\sigma_3^0 = \left(\frac{\mathrm{m}_1^*\xi^2 + p^2}{\mathrm{m}_3^*}\right)\left(\frac{\mathrm{m}_3^*\xi^2 + p^2}{\mathrm{m}_1^*}\right)\ \left(\xi^2 + 4p^2\right) + \frac{\varepsilon_1 p^2 \xi^2}{\mathrm{m}_3^*}\left(\frac{\mathrm{m}_3^*\xi^2 + p^2}{\mathrm{m}_1^*}\right),$$

Thus, the expressions given by equations (3.5)-(3.12) with the help of (3.13)-(3.16) and (3.17) represent the solution of plane strain problem under consideration in the transformed domain using eigen value approach.

4. Inversion of the transforms

To obtain the solution of the problem in the physical domain, we must invert the transforms for three theories that is L-S, G-L and G-N. These expressions are functions of z, the parameters of Laplace and Fourier transforms p and ξ respectively and hence are of the form $\overline{f}(x, z, p)$. To get the function $f(x, z, t)$ in the physical domain, first we invert the Fourier transform using

$$\bar{f}(x, z, p) = \int_{-\infty}^{\infty} \exp(i\xi\, x)\, \tilde{f}(\xi, z, p) d\xi = \frac{1}{\pi}\int_0^{\infty} \left\{\cos(\xi\, x)\, \tilde{f}_e + i \sin(\xi\, x)\, \tilde{f}_0\right\} d\xi$$

The last step in the inversion process is to evaluate the integral in equation (4.1). This was done using Romberg's integration with adaptive step size. This method uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero. The details can be found in Press et al. (1986).

5.Numerical Results and Discussion

Following Eringen [1984], we take the following values of relevant parameters for the case of Magnesium crystal as

$\rho = 1.74\ gm/cm^3$, $j = 0.2\times 10^{-15}\ cm^2$, $\lambda = 9.4\times 10^{11}\ dyne/cm^2$,
$\mu = 4.0\times 10^{11} dyne,$ $K = 1.0\times 10^{11}\ dyne/cm^2$, $C^* = 0.23\ Call/gm^0 C,$
$\gamma = 0.779\times 10^{-4} dyne,$ $\varepsilon = 0.073.$ $K^* = 0.6\times 10^{-2} cal/cm\,sec,$
$T_0 = 23^\circ C,$ $\tau_0 = 6.131\times 10^{-13}\sec,$ $\tau_1 = 8.765\times 10^{-13}\sec$
$h = 1\,cm,$ $z = 1$

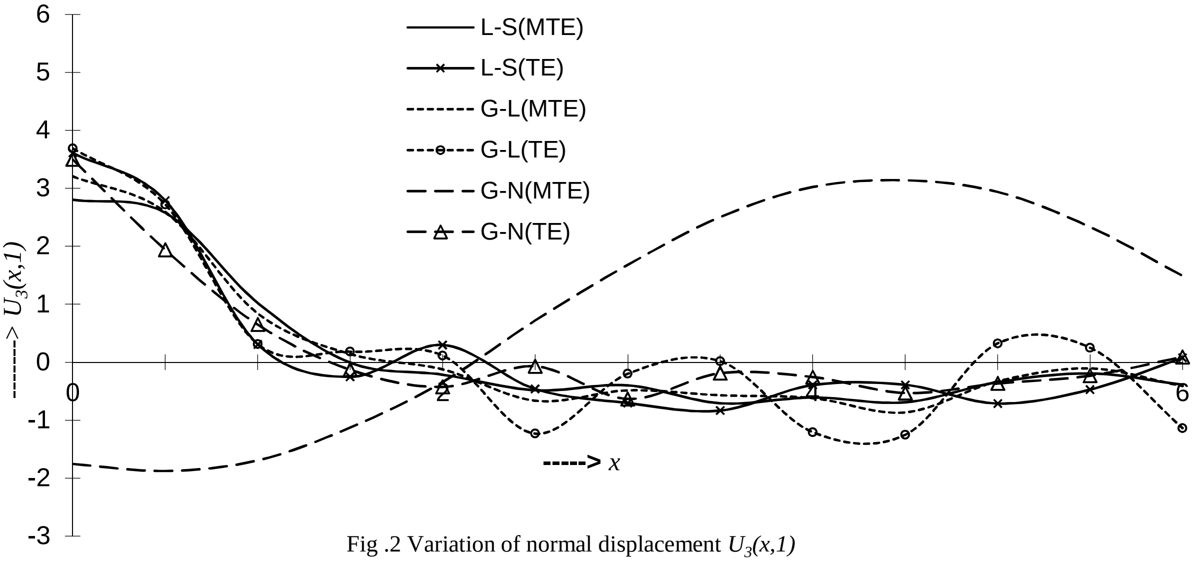


Fig .2 Variation of normal displacement $U_3(x,1)$

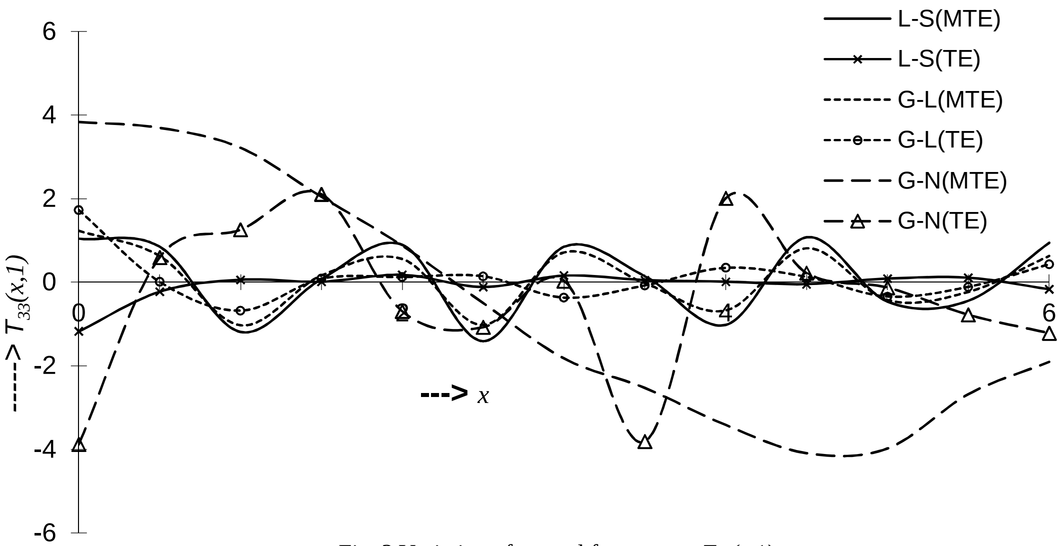


Fig .3 Variation of normal force stress $T_{33}(x,1)$

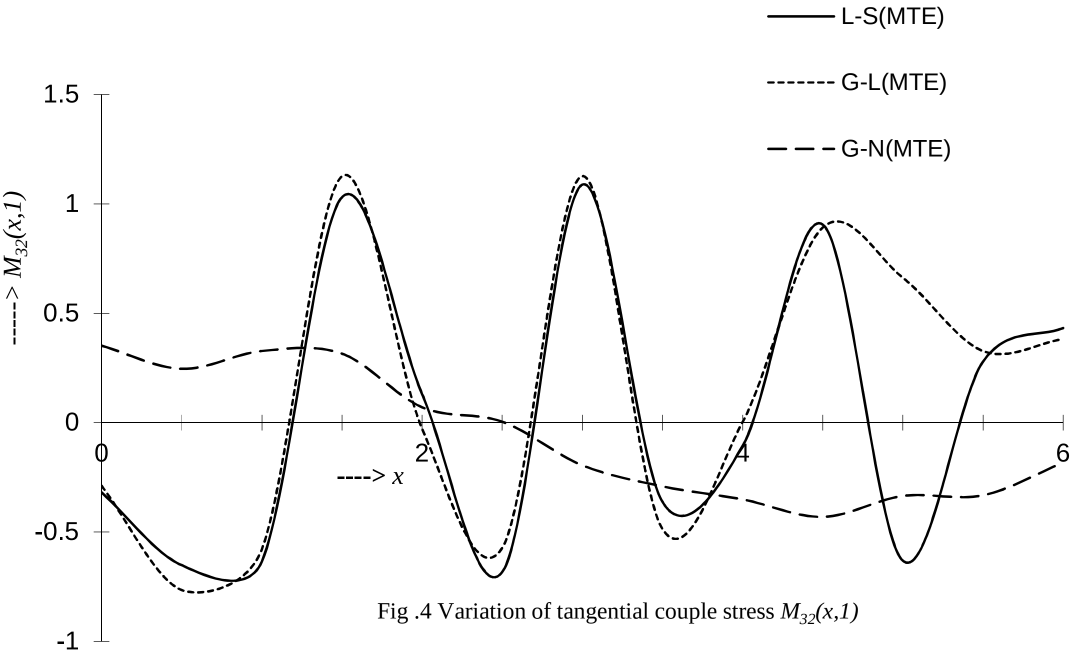


Fig .4 Variation of tangential couple stress $M_{32}(x,1)$

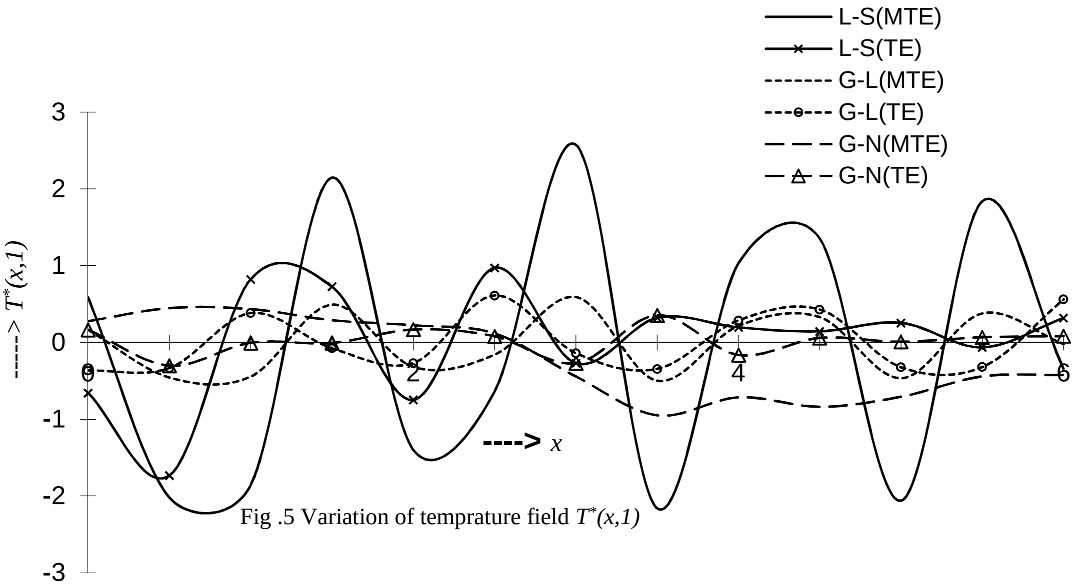


Fig .5 Variation of temprature field $T^*(x,1)$

Discussion

The variations of normal displacement U_3 with distance x for three different theories (L-S, G-L and G-N) in both media after multiplying the original values for G-N theory in MTE medium by

10 are shown in fig. 2 The values of normal displacement due to microrotation effect are less in MTE medium in comparison to TE medium in the $0 \le x \le 0.5$ for all three theories, whereas the values of U_3 oscillate as x increases further in the rest of the range for both media. It is also evident that normal displacement decreases for both media for L-S and G-L theories, increases gradually in MTE medium for G-N theory and oscillate in TE medium for G-N theory.

The values of normal force stress T_{33} in magnitude are more for three different theories in MTE medium in comparison to TE medium. It is also noticed that the values of normal force stress oscillate for L-S and G-L theories in MTE and TE media. The values of normal force stress also oscillate for G-N theory in TE medium, whereas these decrease gradually with increasing value of x in MTE medium. These variations of normal force stress have been shown in fig. 3 after dividing the original values by 10 in case of G-N theory in MTE medium.

Fig. 4 depicts the variations of tangential couple stress M_{32} for three different theories in MTE medium after dividing the original values for G-N theory by 10. The behaviour of tangential couple stress is oscillatory for three theories. It is noticed that the value of tangential couple stress for G-N theory are large in comparison to L-S and G-L theories in the range $0 \le x \le 2.5$ and the values are small for the rest of the range.

The range of values of temperature field in magnitude is large in case of three theories in MTE medium in comparison to TE medium. It is also observed that temperature field oscillate in TE medium for three different theories but in MTE medium for L-S and G-L theories, the temperature field oscillate. The values of temperature field for G-N theory decrease gradually with increasing value of *x* in MTE medium. These variations shown in fig. 5 after multiplying the original values in case of L-S and G-N theories by 10^2 and 10^2 respectively in MTE medium; the original values in case of G-N theory (TE medium) and also magnified by multiplying 10^2.

Conclusion

From the above numerical results, we conclude that micropolarity has a significant effect on normal displacement, normal force stress and temperature field mechanicalsource for three theories.Mcropolar effect is more appreciable for normal displacement and temperature field in, comparison to normal force stress. Application of the present paper may also be found in the field of steel and oil industries. The present Problem is also useful in the field of geomechanics, where, the interest is about the various phenomenon occurring in the earthquakes and measuring of displacements, stresses and temperature field due to the presence of certain sources.

Nomenclature

λ, μ = Lame's constants

α, β ,γ ,K = Micropolar material constants

α_0, λ_0, λ_1 = Material constants due to the presence of stretch.

$\lambda_l, \mu_l, K_l, \alpha_l, v, \gamma_l, \alpha_{0l}, \lambda_{0l}, \lambda_{1l}$ = Microstretch viscoelastic constants

ρ = Density

j = Micro-inertia

u = Displacement vector

$\boldsymbol{\phi}$ = Microrotation vector

$\boldsymbol{\phi}^*$ = Scalar microstretch

t_{ij} = Force stress tensor

m_{ij} = Couple stress tensor

λ_l = Microstress tensor

δ_{ij} = Kronecker delta

ε_{ijr} = Alternating tensor

Δ = Gradient operator

ι = Iota

And dot denotes the partial derivative w.r.t. time.

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