FFP12 2011
Udine 21-23 November 2011

# AN ALTERNATIVE APPROACH TO CANONICAL QUANTIZATION FOR INTRODUCING QUANTUM FIELD THEORY: THE DOUBLE-SLIT EXPERIMENT RE-EXAMINED

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## Abstract
In the last years a growing research concern, within physics education, has been addressed to the production of teaching proposals for introducing notions of quantum field theories (QFT) at the secondary school level. The proposals are usually the result of the effort of translating the most widespread approach to QFT in university textbooks, "canonical quantization", into natural language. After a discussion of the pros and cons of taking canonical quantization as reference, an alternative educational approach is presented. The approach is not yet a teaching proposal but a set of criteria needed for extracting the conceptual and cultural essence of QFT to be taught also without sophisticated formalisms.

> *It is in vain that we say what we see; what we see never resides in what we say. And it is in vain that we attempt to show, by the use of images, metaphors, or similes, what we are saying; the space where they achieve their splendour is not that deployed by our eyes but that defined by the sequential elements of syntax.* (Michel Foucault, The Order of Things)

## 1. Introduction to the research problem
Quantum Field Theory (QFT) is without doubt an advanced and specialized topic that only a part of university physics students is required to deal with. Nevertheless, a growing interest about teaching QFT can be observed in recent years and a certain number of studies exists aimed at producing teaching proposals for introducing notions of QFT at the secondary school level or within introductory physics courses at the university level (see for example Giliberti, 2008). The main motivations of such a growing interest stem from the requirements of secondary school physics curricula towards contemporary physics topics (MIUR, 2010) and from the need of tuning school and extra-school activities: particle physics, the standard model, the last frontiers of physics are indeed object of popular science books and of important and successful exhibitions.
According to an architectural metaphor, QFT can be seen as an imposing building of linguistic engineering, built of very fine materials and assembled by refined craft-like experience; on the other hand, in its usual presentation in university textbooks, it looks like a tangle of threads and formal structures fit one into the other; a 'quarrel' between general-particular, new-old, physical-metaphysical, formal-linguistic aspects.
Entering such a tangle, pointing out its essential conceptual structure, analyzing it from the specific perspective of Physics Education Research (PER), so as to arrive at designing a teaching proposal for secondary school students, is the main goal of the research work we are carrying out.
The specific aims of the present paper are:
- to show that outreach activities and school teaching proposals are usually the results of the effort of translating the most widespread approach currently utilized in university textbooks (canonical quantization) into a familiar language (§2);
- to discuss the pros and cons of the choice of taking 'canonical quantization' as approach of reference for outreach and teaching proposals (§3);
- to present the main features of an alternative approach to QFT that can act as reference for designing outreach activities and school teaching proposals (§4).

## 2. Outreach languages and school teaching proposal on Contemporary Physics
Steven Weinberg, in one of his popularization books on contemporary physics, introduces the notion of elementary particle as follows (Weinberg 1992, p. 25):
*"Furthermore, all these particles are bundle of the energy, or quanta, of various sorts of fields. […] There is one type of field for each species of elementary particles; there is an electron field in the*

*standard model, whose quanta are electrons; there is an electromagnetic field (consisting in electric and magnetic field) whose quanta are photons; [...]"*

Although no explicit formulas appear in the previous explanation, the opacity and technicality of some terms and expressions utilized by Weinberg suggest the idea that a certain formal apparatus underlies the whole argumentation. In particular, statements like these seem to be not only a 'direct translation' of QFT into natural language, but also, more specifically, a direct translation of the most widespread approach to QFT in the university textbooks: the canonical quantization procedure.

The procedure focuses on the electromagnetic field and on the process of quantizing it by replacing, with operators, the numbers representing the coefficients of the Fourier expansion of the solutions of the d'Alembert equation. Every other quantum field (for example, the quantum Dirac field for relativistic spin ½ and m mass particles or the quantum Klein–Gordon (KG) field for relativistic 0 spin and m mass particles) is indeed constructed extending such a formal procedure by analogy: take the generic solution of a wave equation expressed as the Fourier expansion on plane waves, focus on the coefficients of the expansion and elevate them from numbers to operators by defining their commutation (bosons) rules or anti-commutation (fermions) rules. As Mandle & Shaw write: *"From the quantization of the electromagnetic field one is naturally led to the quantization of any classical field, the quanta of the field being particles with well defined properties"* (Mandel & Shaw 1984).

Weinberg's expressions, like canonical quantization:

- provide a paradigmatic role of the electromagnetic field;
- seem to emphasize more the analogy between radiation (electromagnetic field) and matter fields than the differences among them.

Remarkable echoes of the approach described above can be individuated also in the context of PER. The teaching proposal of Art Hobson (Hobson 2005) is a significant example of that. Targeted to secondary school students, such a proposal introduces the notion of "particle as field quanta" by means of the comparison of the two interference patterns created by radiation and matter in the context of the double slit-experiments at low intensity. In particular, after observing the same behaviour between light and matter beam, the author provides the following interpretation:

*"Just as Fig 1.[the interference patter of light at low intensity] is evidence that light is a wave in a field, Fig. 3 [the interference patter of electrons at low intensity] is evidence that matter is a wave in a field – an extended real physical entity that comes through both slits and interferes with itself"*. As far as the relation between matter field and electron is concerned, the author remarks: *"That is, when we say that 'an electron came through double slits,' we really mean that an extended single excited field (space – filling) came through the double – slit" (p.631).*

The key-linguistic expressions used by Hobson ("particle as field quanta", "matter is a wave in a field") are very close to Weinberg's words and, in our opinion, they share with Weinberg the same weaknesses:

i) What do such expressions mean for people or students lacking the formal background on which the reasoning is however grounded?
ii) If the language is taken literally, what does a sentence like "an extended single excited field (space – filling) came through the double-slit" say, when referred to an electron?
iii) Where and how a student is guided to grasp the differences between a classical field and a quantum one?

## 3. Pros and cons of the canonical quantization procedure

The pros of canonical quantization are very evident at university level and concern its technical effectiveness: the formal analogy is a short and direct way to construct the new theoretical entities.

The cons, at least in our opinion, are particularly evident when this approach is taken as a reference for outreach and in designing teaching proposals: in these cases, the lack of formalism on which the procedure is based leads the translation to be conceptually opaque and at risk of giving back a very simplified, semi-classical image of fields and particles.

In particular:

- a students is encouraged to associate to each particle a field whose image is almost the same of the classical electromagnetic field;
- particle and fields are presented as two entities belonging to the same "level" of existence (particles are oscillations of a field coming through both the slits, Hobson).

These two issues are questionable, as well as from the PER perspective, also from the perspective of the Foundations of QFT:
*i)* on the field-particle duality, Haag remarks: "*Yet the belief in a field-particle wave duality as a general principle, the idea that to each particle there is a corresponding field and to each field a corresponding particle has also been misleading and served to veil essential aspects. The role of field is to implement the principle of locality".* (Haag 1996, p. 45-46)
*ii)* on the "different levels", Julian Schwinger, one of the fathers of QFT, underlines: "*Until now, everyone thought that the Dirac equation referred directly to physical particles. Now, in field theory, we recognize that the equations refer to a sublevel. Experimentally we are concerned with particles, yet the old equations describe fields.... When you begin with field equations, you operate on a level where the particles are not there from the start. It is when you solve the field equations that you see the emergence of particles".*

## 4. Alternative approach to Quantum Field Theory

In order to overcome the cognitive, linguistic and philosophical weaknesses of canonical quantization, an alternative educational approach has been built. The approach is not yet a teaching proposal but a set of cultural and operative criteria needed for extracting the conceptual and cultural essence of QFT to be taught also without sophisticated formalisms. In this sense, the construction of the approach represents the preliminary stage needed for preparing the design of a teaching proposal for secondary school students.

### *4.1 Cultural choices*

The cultural choices that characterize the approach are the following:
1. the introduction/discussion of the notions of quantum, quantum field and particle is developed along a *multilevel structure*, i.e. a structure where the levels of phenomenology, formalism and interpretation are related but distinguishable, as suggested by the research on foundations of physics and PER;
2. in the wake of Bohr, the limits of natural (classical) language in translating quantum formalism are taken as a inner feature of contemporary physics and not as a problem to overcome. Familiar images, as well as good metaphors, take indeed their power in bridging the gap between the rich world of experience and the formal physical re-construction of such a world. The gap is in-principle bridgeable when the mathematical reconstruction of the world provides a "projection" of the world itself in a Euclidean space, i.e. when the properties conceptualized by the mathematical description can be re-synthesized in a representation that "lives" in a space somehow "isomorphic" to the space we experience. This game becomes more and more problematic in quantum physics, since the new mathematical description projects the real world in highly abstract, unfamiliar spaces, such as the Hilbert space. "Diagram space" cannot be simply related to "real-world space", and "diagram objects" do not have that set of properties, one is usually led to reassembled in something recognizable as an image of "real objects". As a consequence, the complex relationship between QFT formalism and natural language is assumed to contain a profound cultural value that ought to be exploited by showing the necessary partialness of familiar images or metaphors in giving back how the formalism models quantum objects and their interactions.
3. in order to introduce the notion of quantum field, a *basic* and *essential* conceptual structure is constructed: *basic* means that it does not have to refer to any specific quantum object but it must embody the *common* features of all the quantum objects; *essential* means that it must include *only* what is needed to justify phenomenological evidences. Only in a second stage, the basic and essential structure is *enriched* so as to take into account the "particles zoo". In the logics of searching, first, for what is common to all the quantum objects and, then, for what is specific to the various quantum objects, the phenomenological and conceptual analogy between radiation and matter is presented and where the analogy breaks downs is stressed.

### *4.1 Operative criteria: rough structure of a conceptual path*

The cultural choices of the approach have been implemented in a *rough* conceptual path that should operatively orient the future work aimed at designing a teaching proposal.

The conceptual path foresees two macro-steps.

The first step is based on the operative choice of referring to the double slit experiment at low intensity, as Hobson suggests, as an effective experimental basis for emphasising the analogy in behaviour of radiation and matter and for introducing, from such an analogy, the notions of quantum and quantum field.

The double slit experiment allows both phenomenological constraints to be selected (emission and detection of a quantum object, formation of an interference pattern of the screen, spot by spot), and the "basic and essential conceptual structure" to be constructed for introducing the notion quantum field. The conceptual structure includes: a) time-like and space-like correlations (the quantum object propagates from where it is emitted up to where it is revealed and it is spatially extended) needed for justifying the formation of the interference pattern on the screen; b) the discreetness of dynamical quantities needed for justifying the spots progressively recorded on the screen.

The emphasis on time-like and space-like correlations implies a selection of a few features of a wave-like formalism so as to stress that, when the wave metaphor is used, it cannot be freely extended up to where the common use of the metaphor could lead. For example, it cannot be extended for modelling the superposition of two quantum fields as a classical sum of amplitudes: amplitudes in the formalism of QFT become operators acting on a Fock space.

The focus on the discreetness of dynamical quantities, involved in the interaction with the screen, allows the metaphor of particle to be problematized and discussed within the model of interaction embodied into the theoretical construct of quantum field. This model of interaction is very different from the model expressed by a classical field, like the electromagnetic one. Roughly speaking, the mediation enacted by a classical field between two systems *can* be represented in ordinary three-dimensional space. The transformation of the coefficients of the Fourier expansion of a quantum field into creation and annihilation operators introduces a fundamental change: it introduces the need of referring to an abstract space (the Fock space), where the whole set of possible infinite interaction processes must be explored. In other words, the quantum model of interaction requires a preliminary construction of an abstract "space of possibilities" (represented by Feynman diagrams) that cannot be represented in ordinary space. The model implies hence a fine imaginative game needed for keeping together a not trivial space-time description of the quantum object propagation, the abstract space of all the possible interactions and the physical place where quantum systems are detected. According to our cultural perspective, it would be a pity to short-circuit such a game by using pseudo-familiar images and by "simply" telling that, in quantum interaction, *"the force at distance between two charges is nothing but an exchange of a virtual photon"* (Amaldi 2001, p.473)[1].

In order to stress both the counter-intuitive phenomenological behaviour of the quantum object and the need of taking the phenomenological level apart from the formal/conceptual one, we suggest a specific use of the words *quantum* and *quantum field.* In the wake of Dirac (1930) and Levy-Leblond (2003), we suggest to use the word *quantum* just for fixing the point that the quantum object shows some phenomenological properties (propagation, diffraction/interference and discreteness in interaction) that cannot be all contained in a single image of wave or particle. The word *quantum field* is, instead, used for indicating the formal/theoretical construct able to keep together the properties of *quantum.*

The second macro-step concerns the enrichment of the basic formal structure so as to show where each field gains those properties that allow it to be associated to a specific particle. In order to describe a *particle* it is indeed necessary to move from a model defined by the common properties of radiation and matter to the "particles zoo" constituted by electrons, photons, muons etc. From an operative and experimental point of view, this means to take into account what happens, for

[1] The book of Amaldi is one of the most widespread physics textbooks in Italy for upper secondary school.

example, in the big-science experiments in order to justify the need of providing the notion of quantum with further specifications like mass, electric charge, spin and other quantum numbers (strangeness and so on). From a conceptual point of view, enrichment means to introduce new symmetries (conservation laws) needed to apply the model of interaction, embodied into the notion of quantum field, to the analysis of concrete processes (Hoekzema et al. 2005).
From the more general perspective of the foundations of QFT, the process of enrichment is crucial for pointing out where the analogy between radiation and matter breaks down. Although radiation and matter present common behaviours in the double slit experiments, there is a fundamental difference between them: whilst a classical theory of the electromagnetism does exist, no classical theory of the matter fields exists, as it is formalized by the Gupta-Bleuer condition which is the quantum analogue of the Lorentz condition. Such a constraint makes photons (and the other interaction particles) very different from matter particles.

## 5. Final remarks

In the paper, an approach to QFT alternative to canonical quantization has been presented. Three main elements remark, in our opinion, its peculiarity with respect to canonical quantization:

- instead of playing a prototypical role, the electromagnetic field is addressed as the last step of the process of complexification of the basic structure and the specific feature of the photon of being “an interaction quantum bounded to a classical theory of fields” is stressed;
- the formal analogy, chosen by the canonical quantization as tool for building the different quantum fields on the basis of the electromagnetic one, is replaced by a multi-level structure where the notion of quantum, quantum field and particle are progressively built so as to stress what all the quantum objects have in common (both phenomenologically – the quantum –, and conceptually – the quantum field) and what makes the various objects different (particles);
- instead of stressing mainly the continuity between natural language (or classical images) and QFT formalism, the limits of the former are assumed to contain one of the most important cultural messages to be passed to secondary students.

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