MASS FROM CLASSICAL TO RELATIVISTIC CONTEXT – A PROPOSAL OF CONCEPTUAL UNIFICATION EXPERIMENTED IN THE IDIFO3 SUMMER SCHOOL

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Abstract
The IDIFO3 summer school proposed a basic concept construction in modern physics to young talented students. Inertial and gravitational mass concepts were examined in Newtonian paradigm, the former according to Mach as well. Temporal component of 4-momentum was interpreted as total relativistic energy, through series expansion at low velocities; we then built the relativistic conceptual extension of mass starting from *rest energy*. We integrated an analysis on how students consider mass with an inquiry on mass meanings produced by our path. The results allowed us to recognize 5 student profiles.

1. Why mass?

«The concept of mass is one of the most fundamental notions in physics, comparable in importance only to the concepts of space and time» (Jammer, 2000). The historian of science Max Jammer puts mass on a level with the "backstage" of all physical phenomena: space-time. Mass plays such a founding role because massless macroscopic objects would be made of massless particles, without any form of interaction energy among them: we would have a Universe of free particles travelling at light speed[1]. Mass is to be admitted to endow matter with "something" that characterizes it and that determines its motion in the absence of electromagnetic fields[2].
This quantity has a manifold character. In Newtonian physics, it is essentially the metaphysical "quantity of matter", deeply connected with inertia; the conception is explained in the opening paragraph of *Principia*:

> The quantity of matter is the measurement of the latter obtained from the product of density and volume. […] Air of double density, in a double space, is quadruple in quantity […] And the *norma* [*Latin text*] of all bodies, which be differently condensed for any causes, is identical […]. I will mean hereafter, everywhere, this quantity under the name of body or mass.

Inertial mass acquires its gravitational meaning with the development of the universal gravitation law. In 1883, Mach criticized Newton's conception of mass, because it brought to a vicious circle, and proposed an operative definition through the measure of accelerations. He used a meaningful result derived from Newton's second and third laws, where m_1 can be put as unit of mass:

$$\frac{m_2}{m_1} = \frac{a_1}{a_2}. \quad (1)$$

Eventually, in Special Relativity (SR) a variation of mass corresponds to a variation in the internal energy of a body[3]: the idea of mass becomes more and more subdivided.

2. Learning problems

The concept of mass is very difficult to understand because of this multifaceted character and of the persistence of the ontological vision above as well. A further facet: in the theory of General Relativity, the momentum-energy density modifies the geometry of space-time, so matter exerts an active action on the last, differently from what happened with Newtonian space.
In high-school textbooks, the following learning problems have been found: identification of weight with gravitational mass; belief that a beam balance measures weight; inertial mass defined operationally through *F/a*, where, as a literature example, *F* is the measurable force impressed by a compressed spring. On the subject matter ground, the conception of *quantitas materiae* has also generated misconceptions concerning the mass-energy relation: mass is converted into energy, in a generic sense (6 high-school textbooks); $E=m\,c^2$ represents conversion of mass into energy (1 high-school textbook); confusion/mistakes between energy conservation law and mass conservation law (3 textbooks) [Lehrman, 1982]. In matter of students' learning difficulties, an

[1] We take into account Special Relativity here.
[2] Gamow considers indeed gravity as the force that rules Universe.
[3] This quantity is usually called *rest energy* E_0: the energy of a body measured in its rest reference frame.

experiment on 16 to 18-aged Spanish pupils was performed. They tend to prefer a teleological-qualitative (pre-theoretical) view of mass to an operative-quantitative-formal (scientific) vision. For instance, most of them identified it with other quantities: volume or density on one side and weight on the other; in addition, *quantitas materiae* prevailed on inertia [Domenech, 1993].

3. Relativistic mass debate

In several treatises on relativity a quantity called "relativistic mass" appeared, whose expression is

$$m_r = \frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}. \quad (2)$$

Using the latter, one can rewrite relativistic momentum in the form

$$\mathbf{p} = m_r \mathbf{u} \quad (3)$$

Is it enough to justify the use of (2) as the expression for a proper physical quantity?
Taylor and Wheeler underline that relativistic mass is the first component of relativistic quadrimomentum, while the invariant mass is the magnitude of the latter: a scalar. Actually, why should we interpret *m* in the following master equation as the inertial classical mass?

$$E^2 - p^2c^2 = m^2c^4. \quad (4)$$

In general, there are good reasons both to consider mass in continuity with Newtonian mechanics and to interpret it as a completely new quantity[4] in a new “paradigm” (Kuhn 1970): «the expression $\frac{m_0}{\sqrt{1-\frac{u^2}{c^2}}}$ is best suited for THE mass of a moving body», wrote R. C. Tolman (1934). From the mathematical point of view, the two quantities are completely symmetric: the classical mass can be generalized to two quantities with different tensorial characters (Bickerstaff 1995)[5]. One strong objection to relativistic mass is that

$$m_r = \frac{E}{c^2} \quad (5)$$

varies with the reference frame: it should not define a physical property of a real particle or body. In spite of the last considerations, however, Richard Feynman (1969) claimed the importance of relativistic mass:

> Newton’s Second Law […] was stated with the tacit assumption that *m* is a constant, but we now know that this is not true, and that the mass of a body increases with velocity […]. For those who want to learn just enough about it so they can solve problems, that is all there is to the theory of relativity – it just changes Newton’s laws by introducing a correction factor to the mass.

4. The activity

We took a sample of 42 talented high-school students, aged 17 to 19, selected for attending a modern physics summer school. Our activity consisted in an interactive tutorial with proposals for individual reflections and group discussions. We gave each student worksheets containing open and closed questions; for the discussions, we asked for conceptual differences among the (classical) notions of mass examined first, then what classical meanings of mass underwent a change passing to relativity.
We began the rationale of our path with the quotation in section 1, together with another one containing an operative definition of mass through weight, from *Principia*. An analysis of inertial

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[4] The philosopher of science Feyerabend considered it a «*relation*, involving relative velocities, between an object and a coordinate system», instead of a «*property* of the object itself» (Feyerabend 1965).
[5] Similarly, you can generalize the classical quantity “time” either to the proper time or to the zero component of (*ct, x, y, z)*.

and gravitational mass concepts in Newtonian theory followed, as well as an overview of Mach's definition of the former[6]. We then introduced proper time using light clock, built 4-displacement through analysis of the world lines, and came to quadrimomentum dividing by proper time, in analogy with classic momentum. We finally performed a Taylor series expansion of the temporal component in the Newtonian limit and defined this quantity as relativistic kinetic energy, apart from an additive constant. The identification

$$E_0 = mc^2, \quad (9)$$

as well as its interpretation, were straightforward.

We asked the students some questions after the path:

1) "When does mass plays a role in your everyday life and which phenomena is it involved in?"
2) "What theories of physics do study these phenomena?"
3) "What do you mean when you talk about the *quantity of matter*?"
4) "What connotations and definitions of mass do you know?"
5) "Does the inertial mass of a body change in function of its energy, apart from the kinetic energy?"
6) "In many textbooks *relativistic mass* is mentioned. Explain what it is".

5. Data

Histograms from the collected data follow. For the analysis, we used "vertical" and "horizontal" modalities. The former consists in a separate examination of every answer: we aimed at categorizing them and finding their distribution in the student sample. The latter was a search for each student's own way of "looking at" mass – examining the correlations among answers – to recognize the five profiles, or levels of «physical representation», pointed out by Doménech *et al.*

They are

- ontological (pre-theoretical[7]): mass is either a general property of matter or completely identified with bodies/matter/particles. In this case *quantitas materiae* becomes the best example;
- functional: the quantity is identified with a property, behaviour or trend of a physical system, for instance *inertia* or *heaviness*. An implicit theoretical framework is already present here;
- translational: the quantity is identified with another one whose definition is assumed as well stated, for instance mass as either *density/ volume,* or *weight*;
- relational: the quantity is defined through precise conceptual relationships inside a formal theory made by a number of mathematical laws[8]. In our analysis this representation was reduced to a clear and well-defined outline of *conceptual* relations;
- operational: the quantity is a number that you obtain through a series of explicit and achievable operations; gravitational mass comes out of a measure with an equal arm balance.

Moreover, answers to questions 4 to 6 allowed us to recognize several scientific meanings of mass: inertial, gravitational, Mach's empirical meanings, mass in SR (referred as «energy», «rest energy», «rest mass» by students), "relativistic mass."[9], together with *quantitas materie.*

5.1. *Analysis and Results*

For questions 1 to 3, we divided described phenomena in classes; then physical theories that students join to phenomena were grouped, as well as things described as quantity of matter. The categories are not mutually exclusive.

We found out that in free fall mass is taken into account by a high percentage (17%) of students, showing they haven't understood the physical meaning of the Equivalence Principle.

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[6] We also inserted a reference to mass as the result of interaction with all other masses in the Universe.

[7] It does not refer to a theoretical framework. The definition is then concrete, in spite of appearance: it's implied by a concrete view of physical world.

[8] Mass is given by F/a in the inertial sense and by P/g, or through universal gravitation law as well, in the gravitational sense.

[9] We took these meanings from (Domenech 1993) and extended it to Special Relativity.

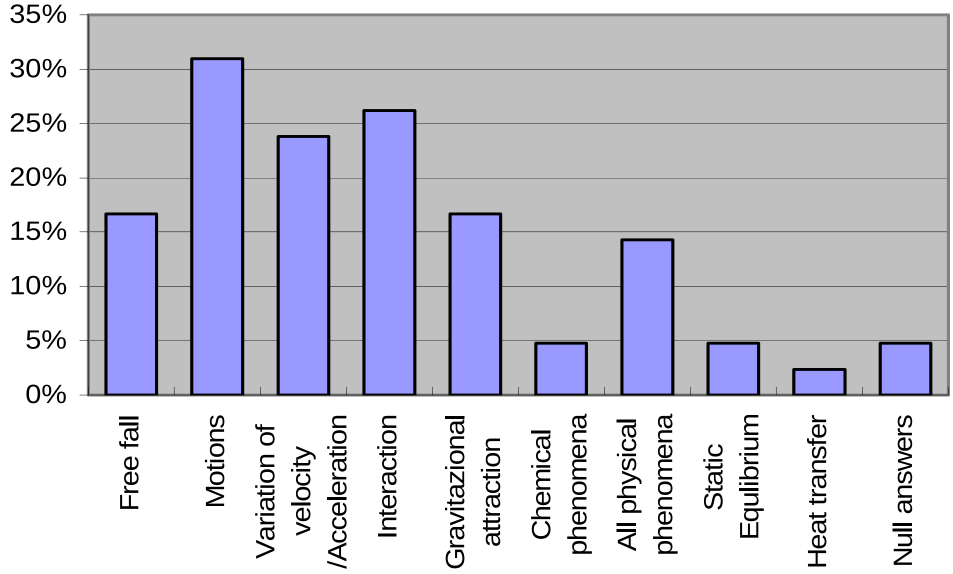

Figure 1: Recalled familiar phenomena, "null answers" stands for answers not concerning the first question

We also found that 24% of the sample considers kinematics as a theory in which mass is a prominent quantity and that 8 students don't distinguish areas from theories as interpretative models for phenomena. From the third question emerges that 32% of students identify correctly "quantity of matter" with mole, both in a general sense (ontological profile, 10%), and as *numbers of moles* (relational profile, 22%). However, identification with mass is present in 15% of the sample.

A meaningful quotation from an answer classified in figure 2 as *quantitas materiae* is «Mass […] can be derived from a formula m = ϱ/V». 45.2% of students only listed mass meanings in these answers.

Only 35/42 students answered to the fifth question. We found no conceptual reference at all to the mass-rest energy equality in 40% of cases. This lack is associated in several cases (20% of the total) to the presence of the idea of "relativistic mass" in students, in an explicit or implicit form, although we stressed the role of relativistic energy in our rationale. In contrast, the conceptual reference is present in 43% of cases, in implicit form or explained in words. There are relatively many uncertain answers (14%), that consists in enunciations, invocation of a generic mass-energy relation or not understandable sentences: 3 students answered "No, because kinetic energy doesn't affect the rest energy", showing that very likely in their conception rest energy is completely unrelated to (inertial) mass.

Thirty-nine students answered to the last question. The concept of "relativistic mass" appeared integrated in Einsteinian paradigm in 36% of the sample. We could not find, by contrast, this integration in 31% of the students. A meaningful example for the category "mass at relativistic speed" is the following: «That means that mass in motion at very high speed can become energy and vice versa». According to this conception, only at high velocities we enter in the realm of relativity, where mass in motion could turn energy. The correct definition – mass depending on speed – was given instead by more than 15% of students, also using formalism; one among them has directly deduced the formula, with the aim of reducing relativistic expression for momentum to the classical one.[10] These students learned the right concept of "relativistic mass" in the proper theoretical framework.

As regards students' profiles, the relational one is clearly dominant: it affects about 60% of the sample. Notice that there is no operational profile, that only one among students uses a partially translational representation and four students the ontological one.

[10] This is, as we have seen, the crucial issue in the debate on this topic.

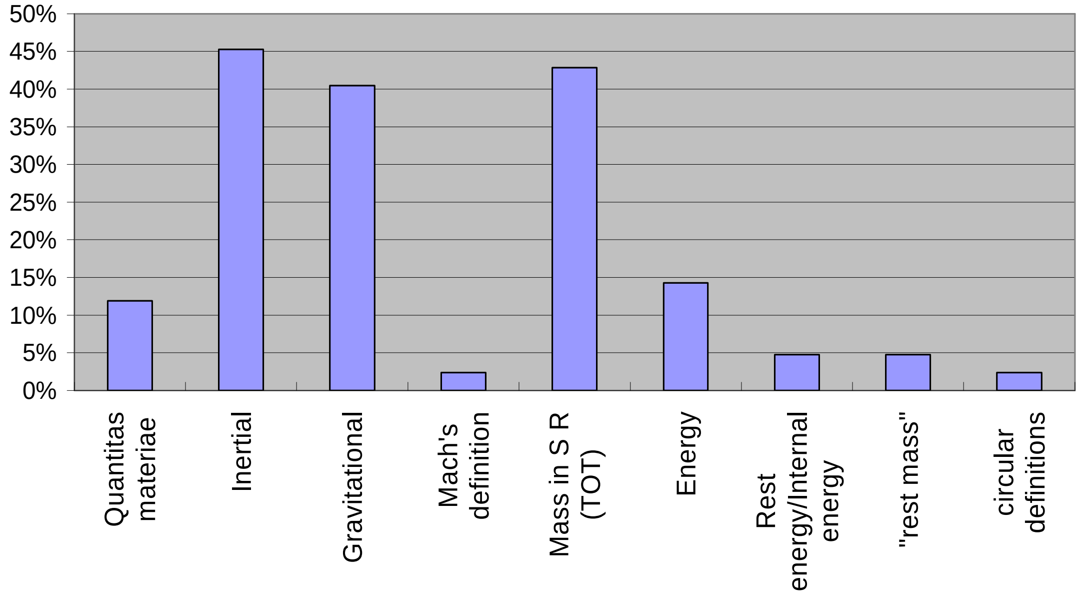

Figure 2: Meanings of the idea of mass (IV question)

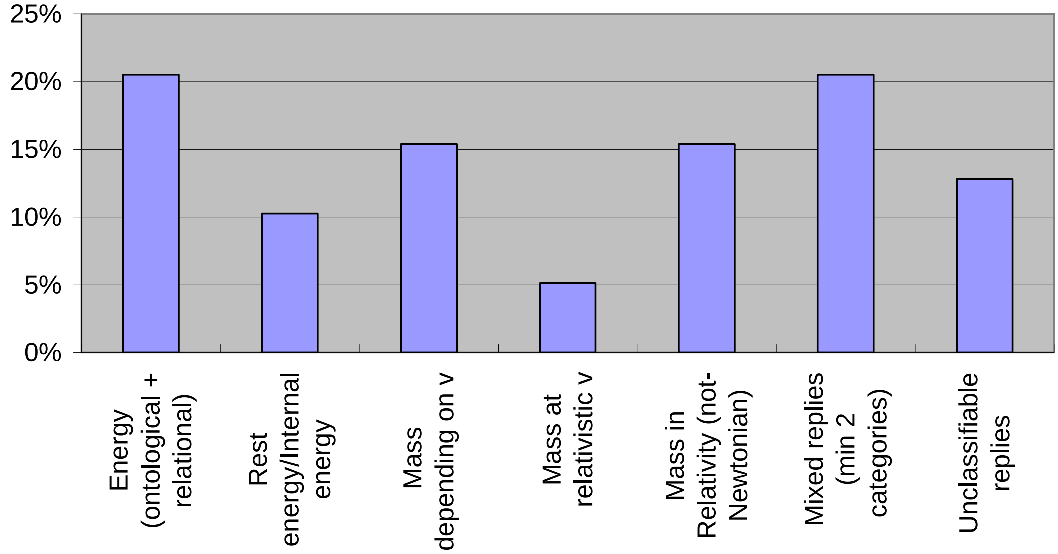

Figure 3: "Relativistic mass" (VI question). The first category includes both a generic relation with energy (ontological level) and a formula connecting E, m, E_0 and m_0 in every possible combination (relational level)

6. Conclusions

Our first research question was "How and in which contexts did our students relate themselves to the term 'mass' and make use of it?" Some interesting elements came out. Phenomena – as well as physical quantities – recalled in familiar semantic areas are in large part mechanical ones, except for quantity of matter, "force fields", "gravitational field". The pre-theoretical conception *quantitas materiae* is rooted in most of students' minds; however, students that answered the first group question never made use of it, as verified in the analysis of video recordings. A few students are aware of the importance of mass in electromagnetism and no one can contextualize it in familiar phenomena. "Ubiquity" is a character of mass expounded by 6 students, but it's not been rationalized by the half of them. In the end, mental representations of mass seem to be strongly affected by learning areas, so it is important to design integrated teaching (Fabri 2007). We noted in particular a local view of the mass in SR in a context defined by speed and a grasping of the concept of mass in SR as limited to a "chapter" of physics.

Our second research question was <u>“How did students interpret the extension of the concept ‘mass’ to ‘mass as rest energy’ in the relativistic context (under the influence of our path)?”</u> It came out that the young talents show very good ability to formalize, the relationship (9) being an important exception in this regard. Besides, <u>terminology</u> plays an important role in the proper understanding of mass in relativity and in framing its conceptual relations with total energy, rest energy, "relativistic mass“. Indeed, wrong answers about the latter, reporting learning difficulties concerning relation (2), have been found in 26% of the students. Moreover, probably misconceptions found about mole were generated by the identity between the word for the physical quantity and for the unit.

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