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BEHAVIOUR AT ULTRA HIGH ENERGIES

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Abstract

The Large Hadron Collider has already attained an unprecedented enery of 7 TeV. By 2013 it is expected to reach its peak energy of double this figure. We can hope that many surprises and discoveries are waiting to happen in the years to come. In this context we explore the behaviour of particles, particularly fermions at these Ultra High Energies. In particular two aspects will be touched upon : The Feshbach-Villars formulation for high energies and also considerations at the Planck length. Some new insights are explored thereby.

1. Introduction

The LHC in Geneva is already operating at a total energy of 7 TeV and hopefully after a pause in 2012, it will attain its full capacity of 14 TeV in 2013. These are the highest energies achieved todate in any accelerator. It is against this backdrop that it is worthwhile to revisit very high energy collisions of Fermions. We will in fact examine their behaviour at such energies.

To get further insight, let us consider the so called Feshbach-Villars formulation (Feshbach and Villars 1958) and analyze the problem from this point of view rather than that of conventional Field theory. In this case with an elementary transformation, the equations for the components φ and χ of the Dirac wave function can be written as

$$i\hbar\left(\partial\phi/\partial t\right) = \left(1/2m\right)\left(\hbar/i\setminus\nabla - eA/c\right)^2\left(\phi+\chi\right)$$
$$\left(e\phi + mc^2\right)\chi$$
$$i\hbar\left(\partial\chi/\partial t\right) = -\left(1/2m\right)\left(\hbar/i\nabla - eA/c\right)^2\left(\phi+\chi\right)$$
$$\left(e\phi - mc^2\right)\phi \qquad (1)$$

What Feshbach and Villars did was give a particle interpretation to the Klein-Gordon and Dirac equations without invoking field theory or the Dirac sea. In this case ϕ represents the "low energy" solutions, that is the normal solution and χ represents the "high energy" solutions. It must be remembered that at our usual energies it is the wave function ϕ, the so called positive energy solution that dominates, χ being of the order of v^2/c^2 of ϕ. On the other hand at very "high energies" χ the so called negative energy solution dominates. Feshbach and Villars identified these two solutions with particles and antiparticles respectively. We have

$$\varphi = \begin{pmatrix}\phi_0 \\ \chi_0\end{pmatrix} e^{i/\hbar(p\times Et)}, \ \phi = \phi_0(p)e^{(i/\hbar(p\times - Et)} \qquad (2)$$

We consider separately the positive and negative values of E (coming from (2)), viz.,

$$E = \pm E_p \ ; \quad E_p = \left[(cp)^2 + \left(mc^2\right)^2\right]^{\frac{1}{2}} \qquad (3)$$

The solutions associated with these two values of E are

$$\phi_0^{(+)} = \frac{E_p + mc^2}{2(mc^2E_p)^{1/2}} \qquad \chi_0^{(+)} = \frac{mc^2 - E_p}{2(mc^2E_p)^{1/2}}$$

$$\phi_0^{(-)} = \frac{mc^2 - E_p}{2(mc^2 E_p)^{1/2}} \qquad \chi_0^{(-)} = \frac{E_p + mc^2}{2(mc^2 E_p)^{1/2}}$$

For $E = \pm E_p$.

As is well known the positive solution $(E = E_p)$ and the negative solution $(E = -E_p)$ represent solutions of opposite charge. We also mention the well known fact that a meaningful subluminal velocity operator can be obtained only from the wave packets formed by positive energy solutions. However the positive energy solutions alone do not form a complete set, unlike in the non relativistic theory. This also means that a point description in terms of the positive energy solutions alone is not possible for the K-G (or the Dirac) equation, that is for the position operator,

$$\delta\left(\vec{X} - \vec{X}_0\right)$$

In fact the eigen states of this position operator include both positive and negative solutions. All this is well known (Cf.ref.(Feshback 1958)).
This matter was investigated earlier by Newton and Wigner too (Newton and Wigner (1949) from a slightly different angle. Some years ago the author revisited this aspect from yet another point of view (Sidharth 2002) and showed that this is symptomatic of noncommutativity which is exhibited by

$$[x_\imath, x_j] = O(l^2)\cdot\Theta_{\imath j}$$

and is related to spin and extension. The noncommutative nature of spacetime has been a matter of renewed interest in recent years particularly in Quantum Gravity approaches. At very high energies, it has been argued that (Sidharth 2008a) there is a minimum fuzzy interval, symptomatic of a non commutative spacetime, so the usual energy momentum relation gets modified and becomes (Sidharth 2008b)

$$E^2 = p^2 + m^2 + \alpha l^2 p^4 \tag{4}$$

the so called Snyder-Sidharth Hamiltonian (Sidharth 2008c, Glinka 2008, Raolina 2011). It has been argued that for fermions $\alpha > 0$ while α can be < 0 for bosons. Using (4) it is possible to deduce the ultrarelativistic Dirac equation (Sidharth 2005a, Sahoo 2010)

$$(D + \beta l p^2 \gamma^5)\psi = 0 \tag{5}$$

$\beta = \sqrt{\alpha}$. In (5) D is the usual Dirac operator while the extra term appears due to the new dispersion relation (4).
As indicated above α is positive. It is known that (Sidharth 2010c), in this case equation (5) can be written in Hamiltonian form

$$-\gamma_0 p^0 \psi = (\bar{D} + \imath \alpha l p^2 \gamma_5)\psi \tag{6}$$

where $\bar{D} \equiv \sum_\imath \gamma^\imath p_\imath$. Further it is well known that the Hamiltonian is given by

$$H = \imath\gamma_5 \overline{\sum}\cdot\vec{p} = \imath\gamma_5 \mid \vec{p} \mid\mid s(\vec{p}) \mid \tag{7}$$

It can be seen from (7) that the Dirac particle acquires an additional mass. However what is very interesting is that the extra mass term is not invariant under parity owing to the presence of γ_5. Indeed as we know from the theory of Dirac matrices

$$P\gamma_5 = -P\gamma_5 \tag{8}$$

In the case of a massless Dirac particle, it was argued that this leads to the mass of the neutrino (Sidharth 2010b).

Thus the mass m gets split into $m+m'$ and $m-m'$ with two states, Ψ_L and Ψ_R. Remembering that a dominant ϕ and a dominant χ respectively represent particle and antiparticle in this Feshbach-Villars formulation and also remembering that under reflection, as is well known,

$$\phi \to \phi, \quad \chi \to -\chi \tag{9}$$

we can see that this means that the particle and antiparticle have different masses, namely $m+m'$ and $m - m'$. Indeed this conclusion was anticipated earlier (Sidharth 2010d). The difference would be minute but in principle can be observed. Already there have been reports of such mass asymmetry being observed in the MINOS Fermi Lab experiment with neutrinos and anti neutrinos (Report 2010). What the MINOS team recorded was a difference in the Δm^2 value for neutrinos and anti neutrinos by as much as forty percent. It is expected that more definitive results would be available by 2012.

It has been pointed out that the fact that equations like (7) and the following applied to neutrinos which are massless suggests one (or more) neutrinos. This is brought out more clearly in the above. Remarkably there seems to be very recent confirmation of such an extra or sterile neutrino (Roe)..

The above discussion brings out ultra high energy effects in Fermionic behavior. Already equation (4) shows modifications to Lorentz symmetry, as has been discussed in detail in several places, for example (Cf.ref.(Sidharth 2008b, Sidharth 2008c) and references therein). This exposes the limits of strict special relativistic considerations.

2. Extra Relativistic Effects

It has just been announced that the OPERA (Oscillation Project with Emulsion Tracking Apparatus) experiment, 1400 meters underground in the Gran Sasso National Laboratory in Italy has detected neutrinos travelling faster than the speed of light, which has been a well acknowledged speed barrier in physics. This limit is 299792,458 meters per second, whereas the experiment has detected a speed of 299,798,454 meters per second. In this experiment neutrinos from the CERN Laboratory 730 kilometers away in Geneva were observed. They arrived 60 nano seconds faster than expected, that is faster than the time allowed by the speed of light. The experiment has been measured to 6σ level of confidence, which makes it a certainty (Adam) and has been repeated again. However it is such an astounding discovery that the OPERA scientists would like further confirmation from other parts of the world. In 2007 the MINOS experiment near Chicago did find hints of this superluminal effect.

It must be reported that the author had predicted such deviations from Einstein's Theory of Relativity, starting from 2000 (Cf.eq.(4)).

(4) shows that the energy at very high energies for fermions is greater than that given by the relativity theory so that effectively the speed of the particle is slightly greater than that of light. For example, if in the usual formula, we replace c by $c+c'$, then, comparing with the above we would get:

$$c' = \alpha l^2 p^4 \cdot [4m^2c^3 + 2p^2c]^{-1}$$

The difference is slight, but as can be seen is maximum for the lightest fermions, viz., neutrinos which are in any case already travelling with the velocity c. We could also argue that the extra term in the Snyder-Sidharth Hamiltonian can contribute partly to an oscillating mass of the neutrino oscillations and partly to a fluctuating super luminal velocity.

3. Ultra High Energy Particles

Let us look at all this differently. Following Weinberg (Weinberg 1972) let us suppose that in one reference frame S an event at x_2 is observed to occur later than one at x_1, that is, $x_2^0 > x_1^0$ with usual notation. A second observer S' moving with relative velocity $\vec{v}$ will see the events separated by a time difference

$$x_2^0 - x_1^0 = \Lambda_\alpha^0(v)(x_2^\alpha - x_1^\alpha)$$

where $\Lambda_\alpha^\beta(v)$ is the "boost" defined by or,

$$x_2^{'0} - x_1^{'0} = \gamma(x_2^0 - x_1^0) + \gamma\vec{v}\cdot(x_2 - x_1)$$

and this will be negative if

$$v\cdot(x_2 - x_1) < -(x_2^0 - x_1^0) \quad (10)$$

We now quote from Weinberg (Weinberg 1972):

"Although the relativity of temporal order raises no problems for classical physics, it plays a profound role in quantum theories. The uncertainty principle tells us that when we specify that a particle is at position x_1 at time t_1, we cannot also define its velocity precisely. In consequence there is a certain chance of a particle getting from x_1 to x_2 even if $x_1 - x_2$ is spacelike, that is, $|x_1 - x_2| > |x_1^0 - x_2^0|$. To be more precise, the probability of a particle reaching x_2 if it starts at x_1 is nonnegligible as long as

$$(x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 \leq \frac{\hbar^2}{m^2} \quad (11)$$

where $\hbar$ is Planck's constant (divided by 2π) and m is the particle mass. (Such space-time intervals are very small even for elementary particle masses; for instance, if m is the mass of a proton then $\hbar / m = w \times 10^{-14} cm$ or in time units $6 \times 10^{-25} sec.$ Recall that in our units $1 sec = 3 \times 10^{10} cm)$. We are thus faced again with our paradox; if one observer sees a particle emitted at x_1, and absorbed at x_2, and if $(x_1 - x_2)^2 - (x_1^0 - x_2^0)^2$ is positive (but less than $\hbar^2 / m^2$), then a second observer may see the particle absorbed at x_2 at a time t_2 before the time t_1 it is emitted at x_1".

To put it another way, the temporal order of causally connected events cannot be inverted in classical physics, but in Quantum Mechanics, the Heisenberg Uncertainty Principle leaves a loop hole.

As can be seen from the above, the two observers S and S' see two different events, viz., one sees, in this example the protons while the other sees neutrons. Moreover, this is a result stemming from (11), viz.,

$$0 < (x_1 - x_2)^2 - (x_1^0 - x_2^0)^2 (\leq \frac{\hbar^2}{m^2}) \quad (12)$$

The inequality (12) points to a reversal of time instants (t_1, t_2) as noted above. However, as can be seen from (12), this happens within the Compton wavelength.

We now observe that in the above formulation for the wave function

$$\psi = \begin{pmatrix} \phi \\ \chi \end{pmatrix},$$

where, as noted, ϕ and χ are, for the Dirac equation, each two spinors. ϕ (or more correctly ϕ_0) represents a particle while χ represents an antiparticle. So, for one observer we have

$$\chi \approx 0 \tag{13}$$

and for another observer we can have

$$\phi \approx 0 \tag{14}$$

that is the two observers would see respectively a particle and an antiparticle. This would be the same for a single observer, if for example the particle's velocity got a boost so that (14) rather than (13) would dominate after sometime.

Interestingly, just after the Big Bang, due to the high energy, we would expect, first (14) that is antiparticles to dominate, then as the universe rapidly cools, particles and antiparticles would be in the same or similar number as in the Standard Model, and finally on further cooling (13) that is particles or matter would dominate.

Finally we now make two brief observations, relevant to the above considerations. Latest results in proton-antiproton collisions at Fermi Lab have thrown up the Bs mesons which in turn have decayed exhibiting CP violations in excess of the predictions of the Standard Model, and moreover this seems to hint at a new rapidly decaying particle. Furthermore, in these high energy collisions particle to antiparticle and vice versa transformations have been detected.

4. Ultra High Energy Particles

Some years ago (Sidharth 2006), we explored some intriguing aspects of gravitation at the micro and macro scales. We now propose to tie up a few remaining loose ends. At the same time, this will give us some insight into the nature of gravitation itself and why it has defied unification with other interactions for nearly a century. For this, our starting point is an array of n Planck scale particles.

As discussed in detail elsewhere, such an array would in general be described by (Jack Ng and Van Dam (1994)

$$l = \sqrt{n\Delta x^2} \tag{15}$$

$$ka^2 \equiv k\Delta x^2 = \frac{1}{2}k_B T \tag{16}$$

where k_B is the Boltzmann constant, T the temperature, r the extent and k is the analogues of the spring constant given by

$$\omega_0^2 = \frac{k}{m} \tag{17}$$

$$\omega = \left(\frac{k}{m}a^2\right)^{\frac{1}{2}} \frac{1}{r} = \omega_0 \frac{a}{r} \tag{18}$$

We now identify the particles with Planck masses and set $\Delta x \equiv a = l_P$, the Planck length. It may be immediately observed that use of (17) and (16) gives $k_B T \sim m_P c^2$, which ofcourse agrees with the temperature of a black hole of Planck mass. Indeed, Rosen (Rosen 1993) had shown that a Planck mass particle at the Planck scale can be considered to be a Universe in itself with a Schwarzchild radius equalling the Planck length.

Whence the mass of the array is given by

$$m = m_P / \sqrt{n} \tag{19}$$

while we have,

$$l = \sqrt{n} l_P, \tau = \sqrt{n} \tau_P, \quad (20)$$

$$l_P^2 = \frac{\hbar}{m_P} \tau_P$$

In the above $m_P \sim 10^{-5} gms, l_P \sim 10^{-33} cm$ and $\tau_P \sim 10^{-42} sec$, the original Planck scale as defined by Max Planck himself. We would like the above array to represent a typical elementary particle. Then we can characterize the number n precisely. For this we use in (19) and (20)

$$l_P = \frac{2`Gm_P}{c^2} \quad (21)$$

which expresses the well known fact that the Planck length is the Schwarzchild radius of a Planck mass black hole, following Rosen. This gives

$$n = \frac{lc^2}{Gm} \sim 10^{40} \quad (22)$$

where l and m in the above relations are the Compton wavelength and mass of a typical elementary particle and are respectively $\sim 10^{-12} cms$ and $10^{-25} gms$ respectively.
Before coming to an interpretation of these results we use the well known result alluded to that the individual minimal oscillators are black holes or mini Universes as shown by Rosen (Rosen 1993). So using the Beckenstein temperature formula for these primordial black holes (Ruffini and Zang 1983), that is

$$kT = \frac{\hbar c^3}{8\pi Gm}$$

we can show that

$$Gm^2 \sim \hbar c \quad (23)$$

We can easily verify that (23) leads to the value $m = m_P \sim 10^{-5} gms$. In deducing (23}) we have used the typical expressions for the frequency as the inverse of the time – the Compton time in this case and similarly the expression for the Compton length. However it must be reiterated that no specific values for l or m were considered in the deduction of (23).
We now make two interesting comments. Cercignani and co-workers have shown (Cercignani 1998, Cercignani 1972) that when the gravitational energy becomes of the order of the electromagnetic energy in the case of the Zero Point oscillators, that is

$$\frac{G\hbar^2\omega^3}{c^5} \sim \hbar\omega \quad (24)$$

then this defines a threshold frequency ω_{max} above which the oscillations become chaotic. In other words, for meaningful physics we require that

$$\omega \leq \omega_{max}.$$

where ω_{max} is given by (24). Secondly as we can see from the parallel but unrelated theory of phonons (Huang 1975, Reif 1965), which are also bosonic oscillators, we deduce a maximal frequency given by

$$\omega_{max}^2 = \frac{c^2}{l^2} \quad (25)$$

In (25) c is, in the particular case of phonons, the velocity of propagation, that is the velocity of sound, whereas in our case this velocity is that of light. Frequencies greater than ω_{max} in (25) are again meaningless. We can easily verify that using (24) in (25) gives back (23). As $\hbar c = 137 e^2$, in a Large Number sense, (23) can also be written as,

$$Gm_P^2 \sim e^2$$

That is, (23) expresses the known fact that at the Planck scale, electromagnetism equals gravitation in terms of strength.
In other words, gravitation shows up as the residual energy from the formation of the particles in the universe via Planck scales particles.
The scenario which emerges is the following. Analogous to Prigogine cosmology (Prigogine 1996, Tryon 1973), from the dark energy background, in a phase transition Planck scale particles are suddenly created. These then condense into the longer lived elementary particles by the above process of forming arrays. But the energy at the Planck scales manifests itself as gravitation, thereafter.
We will further discuss this in the next section.

5. Discussion

Equation (22) can also be written as

$$\frac{Gm}{lc^2} \sim \sqrt{N} \tag{26}$$

where $N \sim 10^{80}$ is the Dirac Large Number, viz., the number of particles in the universe. There are two remarkable features of (22) or (26) to be noted. The first is that it was deduced as a consequence in the author's 1997 cosmological model (Sidharth 2005b). In this case, particles are created fluctuationally from the background dark energy. The model predicted a dark energy driven accelerating universe with a small cosmological constant. It may be recalled that at that time the prevailing paradigm was exactly opposite -- that of a dark matter constrained decelerating universe.
As is now well known, shortly thereafter this new dark energy driven accelerating universe with a small cosmological constant was confirmed conclusively through the observations of distant supernovae. It may be mentioned that the model also deduced other inexplicable relations like the Weinberg formula that relates the microphysical constants with a large scale parameter like the Hubble Constant:

$$m \approx \left(\frac{H\hbar^2}{Gc}\right)^{\frac{1}{3}} \tag{27}$$

While (27) has been loosely explained away as an accidental coincidence Weinberg (Weinberg 1972) himself emphasized that the mysterious relation is in fact unexplained. To quote him, "In contrast (this) relates a single cosmological parameter (the Hubble Constant) to the fundamental constants $\hbar, G, c$ and m and is so far unexplained."
The other feature is that (26) like (27) expresses a single large scale parameter viz., the number of particles in the universe or the Hubble constant in terms of purely microphysical parameters.
As we saw the scenario is similar to the Prigogine cosmology in which out of what Prigogine called the Quantum Vacuum, or what today we may call Dark Energy background, Planck scale or Planck mass are created in a phase transition, very similar to the formation of Benard cells (Nicolis and Prigogine 1989). The energy at the Planck scale, given by (24) then gets distributed in the universe -- amongst all the particles, as the Planck particles form these various elementary particles according to equations (15) to (20). This is brought out by the fact that equation (26) can also be written as the well known Eddington formula:

$$Gm^2 / e^2 \sim \frac{1}{\sqrt{N}} \tag{28}$$

which was believed to be another ad hoc coincidence unrelated to (27). Equation (28) shows how the gravitational force over the cosmos is weak compared to the electromagnetic force. In other words the initial "gravitational energy" on the formation of the Planck scale particles, that is (23) is distributed amongst the various particles of the universe (Sidharth 2005c). From this point of view while l, m, c etc. are indeed microphysical constants as Dirac characterized them, G is not. It is related to the Large Scale cosmos through the Dirac Number N of particles in the universe. This

would also explain the Weinberg puzzle: In this case in equation (27), there are the large scale parameters namely G and H on right side of the equation.
Once we recognize this, we can easily see that unlike what was thought previously, the Weinberg formula (27) is in fact the same as the Dirac formula (28). To see this, we use in (27) two well known relations from cosmology (Cf.eg.(Weinberg 1972)), viz.,

$$R \sim \frac{GM}{c^2} \text{and} M = Nm$$

where R is the radius of the universe $\sim 10^{28} cm$, M its mass $\sim 10^{55} gm$ and m is as before the mass of a typical elementary particle. Then (27) will reduce to (28). Thus, there is only one relation -- (27) or (28), and they express the fact that rather than being a microphysical parameter, G rather than representing a fundamental interaction is related to the large scale cosmos via either of these equations.
It must be observed that this conclusion resembles that of Sakharov (Sakharov 1968), for whom Gravitation was a secondary force like elasticity.

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