

Dark energy condensate and vacuum energy

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Abstract

Many candidate models for dark energy are based on the existence of a classical scalar field. In the context of Quantum Field Theory (QFT), we briefly discuss the condensation of such a field from a light quantum scalar field produced by gradual decay of a heavy particle during cosmological time. We obtain the necessary conditions for survival of the condensate in an expanding universe and show that this process is directly related to quantum nature of the field which preserves the coherence of the condensate at cosmological distances. We also suggest a new interpretation of *vacuum energy* in QFT in curved space time which can potentially solve the puzzle of huge deviation of what is considered to be the vacuum energy from observations of dark energy.

1 Introduction

Many alternatives to a cosmological constant have been proposed to explain the accelerating expansion of the Universe. They can be divided to two main groups: modified gravity models and models in which a field - usually a scalar but also in some cases a vector field - is responsible for what is called dark energy. This limited contribution does not allow to go to the details of each category, therefore we only concentrate on the models based on a scalar field, generally called *quintessence* [Ratra & Peebles 1988, Wetterich 1988]. In simplest version of such models the quintessence field ϕ is a very light scalar with a self-interaction potential and no interaction with other components of the Universe. Under special conditions [Steinhardt, *et al.*1999] the dynamics of the field at late times become independent of its initial value and the field approaches to what is called a tracking solution. In this case it varies very slowly with time and its equation of state, defined as:

$$w = \frac{P}{\rho} = \frac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)} \quad (1)$$

approaches $w \gtrsim -1$. Two types of potentials have tracking solutions: $V(\phi) = e^{-\alpha\phi}$, ϕ^{-n} , and polynomials including such terms. In the context of QFT these potentials are non-renormalizable, thus are assumed to be effective potentials.

A simple quintessence model suffers from various shortcomings. One of these problems is the very small mass of ϕ which must be $m_\phi \sim 10^{-32}$ eV $\sim H_0$. More importantly, this model cannot explain what is called *the coincidence problem*, i.e. why dark energy becomes dominant only after galaxy formation. This means that the density fraction of dark energy at the time of matter formation - presumably after inflation and during reheating - had to be $\sim 10^{42}$ times smaller than matter density. The only natural way to explain such an extreme fine tuning is to consider an interaction between dark energy and other components, notable with dark matter [Amendola 2000, Ziaee pour 2000, Ziaee pour 2004]. Moreover, a simple quintessence model is limited to $w > -1$, but for the time being many observations prefer $w \lesssim -1$, although due to measurement errors one cannot yet have a definitive conclusion about the sign of $w + 1$. Interacting dark energy models can explain $w < -1$ without violation of null energy principle because it has been shown [Ziaee pour 2000, Das, *et al.*2006] that when the interaction is ignored, the effective equation of state $w_{eff} < -1$ when the real $w \geq -1$. As for the particle physics view, no particle is isolated and every species has some non-gravitational interaction with other particles.

The study of quintessence models is usually concentrated on the evolution of a classical scalar field without any concern about how such a field can be formed from a quantum field, specially in an expanding universe. In fact considering very small mass of a quintessence field and its very weak interaction to itself and to other particles, one expects that at their production - during inflation, reheating and/or later in the history of the Universe - they simply behave as relativistic particles and have an equation of state $w \sim 1/3$ which is very different from dark energy $w \approx -1$. Nonetheless, we also know that bosonic particles/fields can condensate and form a classical scalar field. We know this process from condense matter where Cooper pairs creates a non-zero expectation value - a condensate - at macroscopic scales, breaks the $U(1)$ symmetry, generates an effective mass for photons, and leads to phenomena such as superconductivity and super-fluidity. The Higgs field - if it exists - has a similar property but at microscopic scales. The formation of Higgs condensate at electroweak energy scale breaks $SU(2) \times U(1)$ and generates mass for leptons and quarks. The formation of a condensate has been studied, see e.g. [Inagaki 1994] for some Higgs models as well as for inflaton [Parker & Zhang 1993]. These studies show that the problem of condensate formation and evolution is quite involved. In the case of dark energy it is even more complicated because one has to take into account the geometry of the expanding Universe and the evolution of other species, specially in the context of interacting quintessence models. The condensation issue is also more important because in contrast to Higgs and inflation, dark energy condensate must be very uniform and homogeneous both spatially and during cosmic time. These properties cannot be obtained trivially and should strongly constrain quintessence models.

In this proceeding we briefly review the technique, issues, and results obtained recently for a simple and generic interacting quintessence model [Ziaeeepour 2010]. We also describe an idea about a modified definition of vacuum energy which can solve the enormous deviation of the value obtained from usual definition in QFT.

2 Dark energy from decay of dark matter

In the context of inflation-reheating models, all constituents of the Universe were produced either during reheating from the decay of inflaton or a curvaton field, or later on from the decay of other species. In [Ziaeeepour 2004] we have studied the decay of a massive long life metastable dark matter with a small branching ratio to a light scalar field. A classical treatment of such a model show that the energy density of the light scalar field from very early times is roughly constant despite the expansion of the Universe, i.e. it behaves very similar to a cosmological constant, see Fig. 1. In contrast to many quintessence models, in this model the self-interaction potential is a simple ϕ^4 polynomial. The scalar field has a $w \sim -1$ for a large range of parameters and is not very sensitive to self-interaction potential because it is mainly the interaction/decay term that control its evolution during cosmic time. Indeed, such a setup has an internal feedback - if the density of dark energy increases, expansion rate increases, reduces the density of dark matter and thereby the rate of production of scalar field from decay of dark matter, thus the density of dark energy decreases. For a metastable dark matter, this stability can last for very long time. In addition, there would be no big ripe in the future because when a large fraction of dark matter decays, the stability of the system breaks, the energy density of dark energy decreases and the Universe becomes matter or radiation dominated, thus the accelerating expansion rate slow downs. Such a model can easily explain the observed slightly negative value of $w + 1$ [Ziaeeepour 2000, Das, *et al.*2006]. Therefore, for studying the condensation of quintessence field we consider this model.

3 Condensation of an interacting dark energy

In condense matter a condensate is defined as a system in which the majority of particles are in their ground state. In quantum field theory when there is no conserved quantum number, such as in

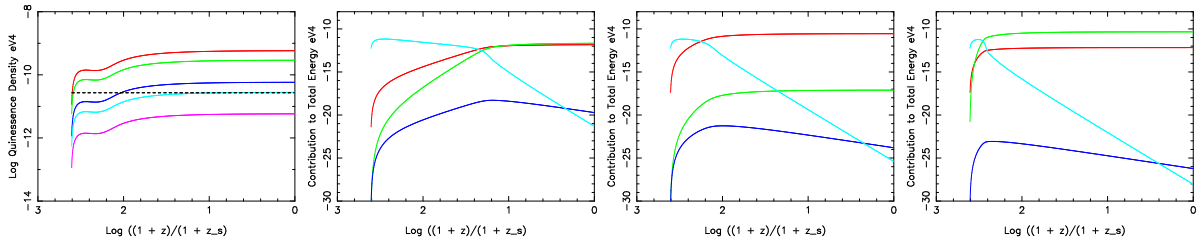


Figure 1: From left to right: 1) Density of dark energy for various branching ratio to the quintessence field $\Gamma_0 \equiv \Gamma_\phi/\Gamma = 10^{-16}$ (magenta), $5\Gamma_0$ (cyan), $10\Gamma_0$ (blue), $50\Gamma_0$ (green), $100\Gamma_0$ (red). Dash line is the observed value of the dark energy. $m_\phi = 10^{-6}eV$, self-coupling $\lambda = 10^{-20}$. 2,3,4) Evolution of the contribution to the total energy density of ϕ for $\Gamma_0 = 10^{-16}$ and 2) $m_\phi = 10^{-8}eV$ and $\lambda = 10^{-20}$; 3) $m_\phi = 10^{-6}eV$ and $\lambda = 10^{-20}$; 4) $m_\phi = 10^{-6}eV$ and $\lambda = 10^{-10}$. Curves are: mass (red), self-interaction (green), kinetic energy (cyan) and interaction with DM (blue).

the case of a single scalar field without internal symmetry and with self interaction, a system is not usually in an eigen state of the number operator. Therefore a condensate which behaves classically is defined as a state in which number operator has a large expectation value - large occupation number - equivalent to a classical system with a large number of particles - in the minimum of their potential energy. Mathematically a condensate state $|\Psi\rangle$ is defined as:

$$\langle\Psi|\phi|\Psi\rangle \equiv \varphi \neq 0 \quad (2)$$

It is easy to verify that in contrast to Bose-Einstein condensate in quantum mechanics, for a quantum system containing only free or weakly interacting - perturbative - fields with finite number of particles φ is zero. It is possible to construct states which satisfies equation (2) using an expansion to coherent states. A special case is suggested by [Matsumoto & Moroi 2008] and a more general state that we call *multi-condensate* is obtained in [Ziaeeepour 2010]. For a real scalar field it has the following expression:

$$|\Psi_{GC}\rangle \equiv \sum_k A_k e^{C_k a_k^\dagger} |0\rangle = \sum_k A_k \sum_{i=0}^{N \rightarrow \infty} \frac{C_k^i}{i!} (a_k^\dagger)^i |0\rangle \quad (3)$$

$$\chi(x, \eta) \equiv a(\eta) \langle\Psi_{GC}|\Phi|\Psi_{GC}\rangle = \sum_k C_k \mathcal{U}_k(x) + C_k^* \mathcal{U}_k^*(x) \implies C_k = \frac{\mathcal{U}_k(x) + \mathcal{U}_k^*(x)}{\chi(x)} \quad (4)$$

where operators a_k and a_k^\dagger are respectively annihilation and creation operators with $[a_k, a_k^\dagger] = 1$. Coefficients A_k and C_k^i are arbitrary, but can depend on the spacetime coordinate because creation and annihilation operators in a curved space depend on the coordinates. \mathcal{U}_k is a solution of the free Green's function of ϕ .

To study the formation and evolution of such a state we consider a toy model for the Universe after reheating. Inspired by the classical model explained in the previous section, we assume a heavy particle - for the sake of simplicity a scalar X - presumably the dark matter, that decays to a light scalar field ϕ and some other particles that we collectively call A . In general ϕ can have a self-interaction considered here to be a simple power-law with positive exponent $V(\phi) = \phi^n$, $n > 0$. For X we consider only a mass term and no self-interaction. The collective field A can have a self-interaction too. The field ϕ can be decomposed to $\phi = \Phi + I\varphi$ with $\langle\Psi|\Phi|\Psi\rangle = 0$, I the unit operator, and $\varphi(x)$ a classical scalar field (a \mathcal{C} -number). According to this definition $\langle\Psi|\phi|\Psi\rangle = \varphi(x)$. After inserting this decomposition to the Lagrangian, the dynamic equation of the classical field can be obtained from variation principle:

$$\frac{1}{\sqrt{-g}} \partial_\mu (\sqrt{-g} g^{\mu\nu} \partial_\nu \varphi) + m_\Phi^2 \varphi + \frac{\lambda}{n} \sum_{i=0}^{n-1} (i+1) \binom{n}{i+1} \varphi^i \langle\phi^{n-i-1}\rangle - \mathbf{g}\langle XA\rangle = 0 \quad (5)$$

A proper solution of this apparently simple equation needs a complete solution of Boltzmann or Kadanov-Baym equations - if we want to consider full nonequilibrium quantum field. This is a very complex problem specially in an expanding universe with a curved spacetime, and needs numerical solution of all the coupled equations. Therefore, rather than considering the full formulation, we assume that the evolution of other components affects φ only through the expansion rate of the Universe $a(t)$ in a FLRW cosmology with metric:

$$ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j = a^2(\eta)(d\eta^2 - \delta_{ij}dx^i dx^j), \quad dt \equiv a d\eta \quad (6)$$

This approximation is applicable to radiation domination and matter domination epochs, but not to redshifts $z \lesssim 1$ when the dark energy becomes dominant. Although the decoupling of $a(t)$ simplifies the problem, we have yet to consider a state, or in classical limit a distribution, for other fields to determine the expectation values in equation (5). Following cosmological observations, we assume a thermal distribution for other components. Evidently this is a valid assumption only at redshifts much smaller than reheating epoch, but due to the long lifetime of X particles only a negligible fraction of them decay earlier and their impact on the later state of matter should be small.

We solve equation (5) separately for radiation and matter domination epochs because they have very different evolution equations. To determine expectation values, we use Schwinger closed time path integral method, but we only consider tree level diagrams and only need to determine free propagators. Considering the very weak coupling of ϕ , this is a good approximation, up to the precision we need here. The Green's function of ϕ is coupled to the classical field φ , but considering the smallness of the self-coupling λ , we first determine the solution without taking into account the coupling term, then we use WKB approximation to obtain a more precise solution. Finally, we use α -vacuum as the initial condition for the Green's function.

During radiation domination epoch the Green's function equation without φ term has an exact solution. The evolution equation for φ also has the same form when interactions, including expectation values, are neglected. Their effect can be added through a WKB approximation. We consider an initial value $\varphi(t_0) = 0$ for the condensate. Finally, we obtain two independent solutions of the evolution equation which are plotted in Fig. 2. As this figure shows, the amplitude of the condensate has an exponential growth, similar to what happens during preheating and resonant decay of inflaton to other fields. This is not a surprise because X and ϕ have a relation analogue to inflaton and matter fields, and have very similar evolution equations. Evidently, the exponential growth of the amplitude cannot continue forever, and backreactions due to nonlinearities in the evolution equation stop the growth rate. In particular, the interaction between the condensate component and *free* ϕ particles through nonlinear self-interaction terms in (5) has the tendency to free particles from condensate, in another word when the density of condensate grows, it begins to *evaporate*. Due to their tiny mass, free particles are relativistic and with the expansion of the Universe they become diluted very quickly. On the other hand, if a large number of them *evaporate*, their energy loss during the expansion of the Universe increases the probability of joining the condensate again. Therefore, as long as the expansion of the Universe is not very quick, this process is self-regulatory. It can be shown that the amplitude of modes decreases very rapidly with increasing $|k|$, i.e. for small distance scales. This is consistent with the lack of significant spatial fluctuation in dark energy density.

In the same way one can solve the Green's function and evolution equations during matter domination epoch. However, even when the interactions are ignored, these equations have a known analytical solution only if $m_\phi = 0$ or $k = 0$. Because we are specially interested in the modes with $|k| \rightarrow 0$ (Similar to radiation domination epoch, it is possible to show that the amplitude of modes for large $|k|$'s decreases quickly), we use the analytical solution for $k = 0$ as zero-order approximation, and apply WKB to obtain a better solution. Finally, we find a solution for the linearized evolution equation of φ which is proportional to $1/\eta$, thus decreases with time. This could be a disastrous for this model, because this leads to a dark energy with $w > -2/3$ which is already ruled out by observations. Nonetheless, when the full nonlinear equation is considered, although we cannot solve it, there is evidence that under special conditions a roughly constant amplitude - a tracking solution

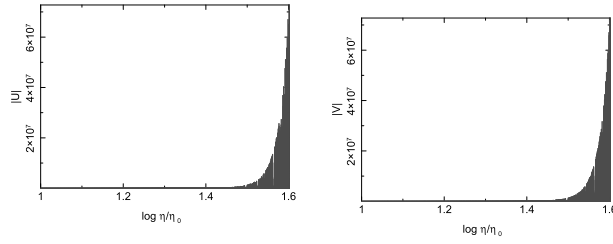


Figure 2: an example of absolute value of independent solutions of evolution function of φ Note that although there are resonant jumps in the solution, due to the complexity of the interaction terms they are not regular like in preheating case.

- similar to observed dark energy can be obtained. In fact, expectation values of type $\langle \phi^i \rangle$ induce negative power of φ into the evolution equation of φ because C_k is proportional to φ^{-1} , see equation (4). As we mentioned in the Introduction, polynomials with negative power are proved to have a tracking solution. A counting of power of self-interaction terms after replacing expectation values with their approximate solution show that for $n \leq 3$ there exists a tracking solution. For $n = 4$ the decrease rate of the condensate density can be enough slow to be consistent with present observations. In 4D spacetimes these potentials are the only renormalizable self-interactions for a quantum scalar field model. Giving the fact that we did not impose any constraint on renormalizability of the model, these results are very interesting and encourage more work on quantum description and origin of dark energy.

If these conclusions are confirmed by a more precise numerical calculations, they would be a proof of the reign of quantum mechanics at largest scales in the Universe because it is the quantum coherence of dark energy that saves it from being diluted by the expansion. The dominance of dark energy at late times in one hand proves that in contrast of general believes, the Universe is dominantly in a coherent quantum state, and in the other hand dark energy provides a natural environment for decoherence of other constituents.

4 Vacuum energy

In QFT energy is calculated as the expectation value of classical expression for the energy momentum tensor $T^{\mu\nu}$ in which classical field is replaced by its quantized counterpart:

$$\begin{aligned}
 E &= \langle \psi | \int d^4k \delta(k_0^2 - m^2) T^{00} | \psi \rangle = \frac{1}{2} \langle \psi | \int d^3\omega_k (a_k^\dagger a_k + a_k a_k^\dagger) | \psi \rangle = \\
 &\langle \psi | \int d^3\omega_k (\hat{N}_k + \frac{1}{2}) | \psi \rangle, \quad \omega_k \equiv \vec{k}^2 + m^2
 \end{aligned}
 \tag{7}$$

Vacuum energy is defined as $|\psi\rangle = |0\rangle$, thus $E_{vac} = \frac{1}{2} \int d^3\omega_k \rightarrow \infty$. To regularize this integral usually a UV cutoff is imposed that leads to a finite but very large value for the energy density of vacuum. In QFT in Minkovsky space without gravity ordering operator is imposed to the above definition i.e. $\langle \psi | : T^{00} : | \psi \rangle$ is used. This simple operation removes the constant (infinite) term and makes a strictly zero vacuum energy. When gravity is present, it is usually supposed that ordering operator cannot be applied because it shifts the energy, an unauthorized operation in the context of general relativity and gravity that define an absolute reference for energy.

The constant term in $\langle \psi | T^{00} | \psi \rangle$ is due to noncommutative creation and annihilation operators in field theory. The presence of the constant term looks like the memory of spacetime or the ghost of a particle, i.e. when a particle is created, its annihilation does not completely restore the initial state. This can be interpreted as a manifestation of energy conservation. In fact, considering two

operators $a^\dagger a$ and aa^\dagger , their physical interpretations are very different. The former counts the number of particles in a state. For an asymptotically free system, it behaves as a detector of particles without changing their state. By contrast, operator aa^\dagger first creates a particle i.e. changes the energy of the system by an amount equal to the energy of the particle. Because in general relativity energy and momentum are locally conserved, this operation necessarily violates the closeness of the system and must actually play the role of a bridge between the system - state - under consideration and another system that provides the energy. Moreover, energy and momentum are eigen values of translation operator. Therefore, the application of right part of this operator changes the translation (symmetry) state of the system. Because symmetry is related to information, the return of energy to its initial reservoir i.e. the annihilation of the particle restore the energy state, but quantum mechanics tells us that it does not restore the information. This is another manifestation of nonlocality or state collapse in quantum mechanics. Nonetheless, if we are only interested in energy conservation, annihilation restores energy-momentum state and in this regard the system should be considered as unchanged. Base on this argument we suggest that operator ordering must be applied to $\langle\psi|T^{00}|\psi\rangle$ even in the context of general relativity.

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