Relativistic Classical and Quantum Mechanics: Clock Synchronization and Bound States, Center of Mass and Particle Worldlines, Localization Problems and Entanglement

Luca Lusanna, Sezione INFN di Firenze, Via Sansone 1, 50019 Sesto Fiorentino (FI), Italy

Abstract: Parametrized Minkowski theories allow to describe isolated systems in global non-inertial frames in Minkowski space-time (defined as M/oller-admissible 3+1 splittings with clock synchronization and radar 4-coordinates) , with the transitions among frames described as gauge transformations. The restriction to the intrinsic inertial rest frame allows to formulate a new relativistic classical mechanics for N-particle systems compatible with relativistic bound states. There is a complete control on the relativistic collective variables (Newton-Wigner center of mass, Fokker-Pryce center of inertia, M/oller center of energy) and on the realization of the Poincare' algebra (with the explicit form of the interaction-dependent Lorentz boosts). The particle world-lines are found to correspond to the ones of predictive mechanics and localization problems are clarified. The model can be consistently quantized avoiding the instantaneous spreading of the center-of-mass wave packets (Hegerfeldt theorem), because the non-local non-covariant center of mass is a non-measurable quantity. The basic difference with non-relativistic quantum mechanics is that the composite N-particle system cannot be represented as the tensor product of single particle Hilbert spaces, but only as the tensor product of the center-of-mass Hilbert space with the one of relative motions. This spatial non-separability (due to the Lorentz signature of space-time) makes relativistic entanglement much more involved than the non-relativistic one. Some final remarks on the emergence of classicality from quantum theory are done.

I. INTRODUCTION

Predictability in classical and quantum physics is possible only if the relevant partial differential equations have a well posed Cauchy problem and the existence and unicity theorem for their solutions applies. A pre-requisite is the existence of a well defined 3-space (i.e. a clock synchronization convention) supporting the Cauchy data. In Galilei space-time there is no problem: time and Euclidean 3-space are absolute.

Instead there is no intrinsic notion of 3-space, simultaneity, 1-way velocity of light (two distant clock are involved) in special relativity (SR): in the absolute Minkowski space-time, only the *conformal structure* (the light-cone) is intrinsically given as the locus of incoming and outgoing radiation. The light postulate says that the 2-way (only one clock is involved) velocity of light c is isotropic and constant. Its codified value replaces the rods (i.e. the standard of length) in modern metrology, where an atomic clock gives the standard of time and a conventional reference frame centered on a given observer is chosen as a standard of space-time (GPS is an example of such a standard).

The standard way out from the problem of 3-space is to choose the Euclidean 3-space of an inertial frame centered on an inertial observer and then use the kinematical Poincaré group to connect different inertial frames. This is done by means of *Einstein convention for the synchronization of clocks*: the inertial observer A sends a ray of light at x_i^o towards the (in general accelerated) observer B; the ray is reflected towards A at a point P of B world-line and then reabsorbed by A at x_f^o; by convention P is synchronous with the mid-point between emission and absorption on A's world-line, i.e. $x_P^o = x_i^o + \frac{1}{2}(x_f^o - x_i^o) = \frac{1}{2}(x_i^o + x_f^o)$. This convention selects the Euclidean instantaneous 3-spaces $x^o = ct = const.$ of the inertial frames centered on A. Only in this case the one-way velocity of light between A and B coincides with the two-way one, c. However, if the observer A is accelerated, the convention breaks down, because if *only* the world-line of the accelerated observer A (the *1+3 point of view*) is given, then the only way for defining instantaneous 3-spaces is to identify them with the Euclidean tangent planes orthogonal to the 4-velocity of the observer (the local rest frames). But these planes (they are tangent spaces not 3-spaces!) will intersect each other at a distance from A's world-line of the order of the acceleration lengths of A, so that all the either linearly or rotationally accelerated frames, centered on accelerated observers, based either on Fermi coordinates or on rotating ones, will develop *coordinate singularities.* Therefore their approximated notion of instantaneous 3-spaces cannot be used for a well-posed Cauchy problem for Maxwell equations.

Therefore in Refs.(Alba and Lusanna, 2007, 2010) a general theory of non-inertial frames in Minkowski space-time was developed. As shown in Refs. (Lusanna, 1997; Alba and Lusanna 2010) this formulation allowed to develop a Lagrangian description (*parametrized Minkowski theories*) of isolated systems (particles, strings, fluids, fields) in which the transition from a non-inertial frame to another one is formalized as a gauge transformation. Therefore the inertial effects only modify the appearances of phenomena but not the physics.

Then the restriction to inertial frames allowed to define the Poincaré generators starting from the energy-momentum tensor. The intrinsic inertial rest frame of every isolated system allowed the definition of the *rest-frame instant form of dynamics* where it is possible to study the problem of the separation of the collective relativistic variable of an isolated system from the relative ones living in the so called Wigner 3-spaces in accord with the theory of relativistic bound states (Alba, Crater and Lusanna, 2010, 2001; Crater and Lusanna, 2001). The ordinary world-lines of the particles can then be reconstructed (Alba, Crater and Lusanna, 2007).

The rest-frame instant form allows to give a formulation of *relativistic quantum mechanics* (RQM) (Alba, Crater and Lusanna,2011) which takes into account of the problems of relativistic causality and relativistic localization implied by the Lorentz signature of Minkowski space-time. Also a definition of relativistic entanglement can be given: the Lorentz signature implies features of *non-locality and spatial non-separability* absent in the non-relativistic formulation of entanglement.

Elsewhere we extend the definition of non-inertial frames to general relativity (GR) in asymptotically Minkowskian space-times. In GR there is no absolute notion since also space-time becomes dynamical (with the metric structure satisfying Einstein's equations): however in this class of space-times it is possible to make a Hamiltonian formulation and to separate the inertial degrees of freedom of the gravitational field from the tidal ones (the gravitational waves of the linearized theory).

II. NON-INERTIAL FRAMES IN MINKOWSKI SPACE-TIME AND RADAR 4-COORDINATES

A metrology-oriented description of non-inertial frames in SR can be done with the *3+1 point of view* and the use of observer-dependent Lorentz scalar *radar 4-coordinates*. Let us give the world-line $x^\mu(\tau)$ of an arbitrary time-like observer carrying a standard atomic clock: τ is an arbitrary monotonically increasing function of the proper time of this clock. Then we give an admissible 3+1 splitting of Minkowski space-time, namely a nice foliation with space-like instantaneous 3-spaces Σ_τ: it is the mathematical idealization of a protocol for clock synchronization. All the clocks in Σ_τ sign the same time of the atomic clock of the observer: it is the non-factual idealization required by the Cauchy problem generalizing the existing protocols for building coordinate system inside the future light-cone of a time-like observer. On each 3-space Σ_τ we choose curvilinear 3-coordinates σ^r having the observer as origin. These are the *radar 4-coordinates* $\sigma^A = (\tau; \sigma^r)$.

If $x^\mu \mapsto \sigma^A(x)$ is the coordinate transformation from the Cartesian 4-coordinates x^μ of a reference inertial observer to radar coordinates, its inverse $\sigma^A \mapsto x^\mu = z^\mu(\tau, \sigma^r)$ defines the *embedding* functions $z^\mu(\tau, \sigma^r)$ describing the 3-spaces Σ_τ as embedded 3-manifold into Minkowski space-time. The induced 4-metric on Σ_τ is the following functional of the embedding ${}^4g_{AB}(\tau, \sigma^r) = [z^\mu_A\, \eta_{\mu\nu}\, z^\nu_B](\tau, \sigma^r)$, where $z^\mu_A = \partial\, z^\mu / \partial\, \sigma^A$ and ${}^4\eta_{\mu\nu} = \epsilon\, (+---)$ is the flat metric ($\epsilon = \pm 1$ according to either the particle physics $\epsilon = 1$ or the general relativity $\epsilon = -1$ convention). While the 4-vectors $z^\mu_r(\tau, \sigma^u)$ are tangent to Σ_τ, so that the unit normal $l^\mu(\tau, \sigma^u)$ is proportional to $\epsilon^\mu{}_{\alpha\beta\gamma}\, [z^\alpha_1\, z^\beta_2\, z^\gamma_3](\tau, \sigma^u)$, we have $z^\mu_\tau(\tau, \sigma^r) = [N\, l^\mu + N^r\, z^\mu_r](\tau, \sigma^r)$ with $N(\tau, \sigma^r) = \epsilon\, [z^\mu_\tau\, l_\mu](\tau, \sigma^r)$ and $N_r(\tau, \sigma^r) = -\epsilon\, g_{\tau r}(\tau, \sigma^r)$ being the lapse and shift functions.

The foliation is nice and admissible if it satisfies the conditions:
1) $N(\tau, \sigma^r) > 0$ in every point of Σ_τ (the 3-spaces never intersect, avoiding the coordinate singularity of Fermi coordinates);
2) $\epsilon\, {}^4g_{\tau\tau}(\tau, \sigma^r) > 0$, so to avoid the coordinate singularity of the rotating disk, and with the positive-definite 3-metric ${}^3g_{rs}(\tau, \sigma^u) = -\epsilon\, {}^4g_{rs}(\tau, \sigma^u)$ having three positive eigenvalues (these are the Møller conditions);
3) all the 3-spaces Σ_τ must tend to the same space-like hyper-plane at spatial infinity (so that there are always asymptotic inertial observers to be identified with the fixed stars).

These conditions imply that *global rigid rotations are forbidden in relativistic theories*. In Ref.(Alba and Lusanna, 2010) there is the expression of the admissible embedding corresponding to a 3+1 splitting of Minkowski space-time with parallel space-like hyper-planes (not equally spaced due to a linear acceleration) carrying differentially rotating 3-coordinates without the coordinate singularity of the rotating disk. It is the first consistent global non-inertial frame of this type.

Each admissible 3+1 splitting of space-time allows to define two associated congruences of time-like observers: a) the Eulerian observers with the unit normal to Σ_τ as 4-velocity; b) the non surface forming observers with 4-velocity proportional to $z^\mu_\tau(\tau, \sigma^r)$.

Therefore starting from the four independent embedding functions $z^\mu(\tau, \sigma^r)$ we obtain the ten components ${}^4g_{AB}(\tau, \sigma^u)$ of the 4-metric: they play the role of the *inertial potentials* generating the *relativistic apparent forces* in the non-inertial frame (the usual Newtonian inertial potentials can be recovered by doing the non-relativistic limit). The extrinsic curvature tensor ${}^3K_{rs}(\tau, \sigma^u) = [\frac{1}{2N}\, (N_{r|s} + N_{s|r} - \partial_\tau\, {}^3g_{rs})](\tau, \sigma^u)$, describing the *shape* of the instantaneous 3-spaces as embedded 3-sub-manifolds of Minkowski space-time, is a secondary inertial potential functional of the inertial potentials ${}^4g_{AB}$.

III. CLASSICAL RELATIVISTIC ISOLATED SYSTEMS

In these global non-inertial frames of Minkowski space-time it is possible to describe isolated systems (particles, strings, fields, fluids) admitting a Lagrangian formulation by means of *parametrized Minkowski theories*. The matter variables are replaced with new ones knowing the clock synchronization convention defining the 3-spaces Σ_τ. For instance a Klein-Gordon field $\tilde{\phi}(x)$ will be replaced with $\phi(\tau, \sigma^r) = \tilde{\phi}(z(\tau, \sigma^r))$; the same for every other field. Instead for a relativistic particle with world-line $x^\mu(\tau)$ we must make a choice of its energy sign: then the positive- (or negative-) energy particle will be described by 3-coordinates $\eta^r(\tau)$ defined by the intersection of its world-line with Σ_τ: $x^\mu(\tau) = z^\mu(\tau, \eta^r(\tau))$. Differently from all the previous approaches to relativistic mechanics, the dynamical configuration variables are the 3-coordinates $\eta^r(\tau)$ and not the world-lines $x^\mu(\tau)$ (to rebuild them in an arbitrary frame we need the embedding defining that frame!).

Then the matter Lagrangian is coupled to an external gravitational field and the external 4-metric is replaced with the 4-metric $g_{AB}(\tau, \sigma^r)$ of an admissible 3+1 splitting of Minkowski space-time. With this procedure we get a Lagrangian depending on the given matter and on the embedding $z^\mu(\tau, \sigma^r)$, which is invariant under *frame-preserving diffeomorphisms*. As a consequence, there are four first-class constraints (an analogue of the super-Hamiltonian and super-momentum constraints of canonical gravity) implying that the embeddings $z^\mu(\tau, \sigma^r)$ are *gauge variables*, so that *all the admissible non-inertial or inertial frames are gauge equivalent*, namely physics does *not* depend on the clock synchronization convention and on the choice of the 3-coordinates σ^r. Even if the gauge group is formed by the frame-preserving diffeomorphisms, the matter energy-momentum tensor allows the determination of the ten conserved Poincare' generators P^μ and $J^{\mu\nu}$ (assumed finite) of every configuration of the system (in non-inertial frames they are asymptotic generators at spatial infinity like the ADM ones in GR).

If we restrict ourselves to inertial frames, we can define the *inertial rest-frame instant form of dynamics for isolated systems* by choosing the 3+1 splitting corresponding to the intrinsic inertial rest frame of the isolated system centered on an inertial observer: the instantaneous 3-spaces, named *Wigner 3-spaces* due to the fact that the 3-vectors inside them are Wigner spin-1 3-vectors, are orthogonal to the conserved 4-momentum P^μ of the configuration. The embedding corresponding to the inertial rest frame is $z^\mu(\tau, \vec{\sigma}) = Y^\mu(\tau) + \epsilon^\mu_r(\vec{h})\, \sigma^r$, where $Y^\mu(\tau)$ is the Fokker-Pryce center-of-inertia 4-vector, $\vec{h} = \vec{P}/\sqrt{\epsilon\, P^2}$ and ${\epsilon^\mu}_{A=\nu}(\vec{h})$ is the standard Wigner boost for time-like orbits sending $P^\mu = \sqrt{\epsilon\, P^2}\, (\sqrt{1 + \vec{h}^2}; \vec{h})$ to $(1; 0)$.

In Ref.(Alba and Lusanna, 2010) there is also the definition of the admissible *non-inertial rest frames*, where P^μ is orthogonal to the asymptotic space-like hyper-planes to which the instantaneous 3-spaces tend at spatial infinity. This non-inertial family of 3+1 splittings is the only one admitted by the asymptotically Minkowskian space-times without super-translations in GR. Finally in Ref.(Alba, 2006) there is the definition of *parametrized Galilei theories*, non relativistic limit of the parametrized Minkowski theories. Also the inertial and non-inertial frames in Galilei space-time are gauge equivalent in this formulation.

IV. RELATIVISTIC CENTER OF MASS, PARTICLE WORLD-LINES AND BOUND STATES

The framework of the inertial rest frame allowed the solution of the following old open problems:

A) The classification (Alba, Lusanna and Pauri, 2002) of the relativistic collective variables (the canonical non-covariant Newton-Wigner center of mass (or center of spin), the non-canonical covariant Fokker-Pryce center of inertia and the non-canonical non-covariant Møller center of energy), replacing the Newtonian center of mass (and all tending to it in the non-relativistic limit), that can be built only in terms of the Poincare' generators: they are *non measurable* quantities due to the *non-local* character of such generators (they know the whole 3-space Σ_τ). There is a Møller world-tube around the Fokker-Pryce 4-vector containing all the possible pseudo-world-lines of the other two, whose Møller radius $\rho = |\vec{S}|/\sqrt{\epsilon\, P^2}$ ($\vec{S}$ is the rest angular momentum) is determined by the Poincaré Casimirs of the isolated system. This non-covariance world-tube is a non-local effect of Lorentz signature of the space-time absent in Euclidean spaces. The world-lines $x_i^\mu(\tau)$ of the particles are derived (interaction-dependent) quantities and in general they do not satisfy vanishing Poisson brackets (Alba, Crater and Lusanna, 2007): already at the classical level a *non-commutative structure* emerges due to the Lorentz signature!

B) The description of every isolated system as a decoupled (non-measurable) canonical non-covariant (Newton-Wigner) external center of mass (described by frozen Jacobi data) carrying a pole-dipole structure: the invariant mass and the rest spin of the system expressed in terms of suitable Wigner-covariant relative variables of the given isolated system inside the Wigner 3-spaces (after the elimination of the internal center of mass with well defined rest-frame conditions). The invariant mass is the effective Hamiltonian inside these 3-spaces.

C) The formulation of classical relativistic atomic physics (the electro-magnetic field in the radiation gauge plus charged scalar particles with Grassmann-valued electric charges to make a ultraviolet and infrared regularization of the self-energies and with mutual Coulomb potentials) and the identification of the Darwin potential at the classical level by means of a canonical transformation transforming the original system in N charged particles interacting with Coulomb plus Darwin potentials and a free radiation field (absence of Haag's theorem at least at the classical level). Therefore the Coulomb plus Darwin potential is the description as a Cauchy problem of the interaction described by the one-photon exchange Feynman diagram of QED (all the radiative corrections and photon brehmstrahlung are deleted by the Grassmann regularization).

In Ref.(Lusanna, 2011) there are the references for all the interacting systems for which the explicit form of the interaction-dependent Lorentz boosts is known.

V. RELATIVISTIC QUANTUM MECHANICS

In Ref.(Alba, Crater and Lusanna, 2011) there is a new formulation of *relativistic quantum mechanics* in inertial frames englobing all the known results about relativistic bound states due to the use of clock synchronization for the definition of the instantaneous 3-spaces, which implies the absence of relative times in their description.

In Galilei space-time non-relativistic quantum mechanics, where all the main results about entanglement are formulated, describes a composite system with two (or more) subsystems with a Hilbert space which is the tensor product of the Hilbert spaces of the subsystems: $H = H_1 \otimes H_2$. This type of spatial separability is named *the zeroth postulate* of quantum mechanics. However, when the two subsystems are mutually interacting, one makes a unitary transformation to the tensor product of the Hilbert space H_{com} describing the decoupled Newtonian center of mass of the two subsystems and of the Hilbert space H_{rel} of relative variables: $H = H_1 \otimes H_2 = H_{com} \otimes H_{rel}$. This allows to use the method of separation of variables to split the Schroedinger equation in two equations: one for the free motion of the center of mass and another, containing the interactions, for the relative variables (this equation describes both the bound and scattering states). A final unitary transformation of the Hamilton-Jacobi type allows to replace H_{com} with $H_{com,HJ}$, the Hilbert space in which the decoupled center of mass is frozen and described by non-evolving Jacobi data. Therefore we have $H = H_1 \otimes H_2 = H_{com} \otimes H_{rel} = H_{com,HJ} \otimes H_{rel}$.

While at the non-relativistic level these three descriptions are unitary equivalent, this is no more true in RQM. The non-local and non-covariant properties of the decoupled relativistic center of mass, described by the frozen Jacobi data $\vec{z}$ and $\vec{h} = \vec{P}/\sqrt{\epsilon P^2}$, imply that the only consistent relativistic quantization is based on the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$ (in the non-relativistic limit it goes into the corresponding Galilean Hilbert space). We have $H \neq H_1 \otimes H_2$, because, already in the non-interacting case, in the tensor product of two quantum Klein-Gordon fields, $\phi_1(x_1)$ and $\phi_2(x_2)$, most of the states correspond to configurations in Minkowski space-time in which one particle may be present in the absolute future of the other particle. This is due to the fact that the two times x_1^o and x_2^o are totally uncorrelated, or in other words there is no notion of instantaneous 3-space (clock synchronization convention). Also the scalar products in the two formulations are completely different.

We have also $H \neq H_{com} \otimes H_{rel}$, because if instead of $\vec{z} = Mc\,\vec{x}_{NW}(0)$ we use the evolving (non-local and non-covariant) Newton-Wigner position operator $\vec{x}_{NW}(\tau)$, then we get a violation of relativistic causality because the center-of-mass wave packets spread instantaneously as shown by the Hegerfeldt theorem.

Therefore the only consistent Hilbert space is $H = H_{com,HJ} \otimes H_{rel}$. The main complication is the definition of H_{rel}, because we must take into account the three pairs of (interaction-dependent) second-class constraints eliminating the internal 3-center of mass inside the Wigner 3-spaces. When we are not able to make the elimination at the classical level and formulate the dynamics only in terms of Wigner-covariant relative variables, we have to quantize the particle Wigner-covariant 3-variables η_i^r, κ_{ir} and then to define the physical Hilbert space by adding the quantum version of the constraints a la Gupta-Bleuler.

Relativistic quantum mechanics in rotating non-inertial frames by using a multi-temporal quantization scheme is defined in Ref.(Alba and Lusanna, 2006). In it *the inertial gauge variables are not quantized but remain c-numbers*; the known results in atomic and nuclear physics are reproduced.

VI. RELATIVISTIC ENTANGLEMENT AND LOCALIZATION PROBLEMS

Since we have that the Hilbert space $H = H_{com,HJ} \otimes H_{rel}$ is not unitarily equivalent to the one $H_1 \otimes H_2 \otimes ...$, where H_i are the Hilbert spaces of the individual particles, at the relativistic level the zeroth postulate of non-relativistic quantum mechanics does not hold (the necessity to use H_{rel} implies a type of *weak form of relationism* different from the formulations connected to the Mach principle). Since the Hilbert space of composite systems is not the tensor product of the Hilbert spaces of the sub-systems, we need a formulation of *relativistic entanglement* taking into account this spatial non-separability and the non-locality of the center of mass, both coming from the Poincare' group, i.e. from the Lorentz signature of space-time.

Since the center of mass is decoupled, its *non-covariance* is irrelevant. However its *non-locality* is a source of open problems: do we have to quantize it (in the preferred momentum basis implied by the quantum Poincaré algebra) and if yes is it meaningful to consider center-of-mass wave packets or we must add some super-selection rule? The wave function of the non-local center of mass is a kind of wave function of the 3-universe: who will observe it?

As a consequence in SR there are open problems on which type of *relativistic localization* is possible. There are strong indications that the Newton-Wigner position operator *cannot be self-adjoint but only symmetric* with the implication of a bad localization of relativistic particles. The existing problems with relativistic position operators (like the Newton-Wigner one), deriving from the Lorentz signature of Minkowski space-time which forbids the existence of a unique collective variable with all the properties of Newton center of mass , point towards a *non-measurability of absolute positions* (but not of the relative variables needed to describe the spectra of bound states). This type of *un-sharpness* should be induced also in non-relativistic quantum mechanics: in atomic physics the crucial electro-magnetic effects are of order $1/c$. Experiments in atomic and molecular physics are beginning to explore these localization problems, in frameworks dominated by a *Newtonian classical intuition* and taking into account the experimental quantum limits for atom localization.

Therefore SR introduces a *kinematical non-locality* and a *kinematical spatial non-separability*, which reduce the relevance of *quantum non-locality* in the study of the foundational problems of quantum mechanics. Relativistic entanglement will have to be reformulated in terms of relative variables also at the non-relativistic level. Therefore the control of Poincare' kinematics will force to reformulate the experiments connected with Bell inequalities and teleportation in terms of the relative variables of isolated systems containing: a) the observers with their measuring apparatus (Alice and Bob as macroscopic quasi-classical objects); b) the particles of the protocol (but now the ray of light, the "photons" carrying the polarization, move along null geodesics); c) the environment (macroscopic either quantum or quasi-classical object).

VII. OPEN PROBLEMS

The main open problem in SR is the quantization of *fields* in non-inertial frames due to the no-go theorem of Ref.(Torre and Varadarajan, 1999) showing the existence of obstructions to the unitary evolution of a massive quantum Klein-Gordon field between two space-like surfaces of Minkowski space-time. Its solution, i.e. the identification of all the 3+1 splittings allowing unitary evolution, will be a prerequisite to any attempt to quantize canonical gravity taking into account the equivalence principle (global inertial frames do not exist!). Moreover entanglement in non-inertial frames without Rindler observers is still to be formulated.

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