New aspects of collective phenomena at nanoscales in quantum plasmas

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Abstract. We present two novel collective effects is quantum plasmas. First, we discuss a novel attractive force between ions that are shielded by the degenerate electrons in quantum plasmas. Here we show that the electric potential around an isolated ion has a hard core negative part that resembles the Lennard-Jones (LJ)-type potential. Second, we present a theory for stimulated scattering instabilities of electromagnetic waves off quantum plasma modes. Our studies are based on the quantum hydrodynamical description of degenerate electrons that are greatly influenced by electromagnetic and quantum forces. The relevance of our investigation to bringing ions closer for fusion in high-energy solid density plasmas at atomic dimensions, and for producing coherent short wavelength radiation in the x-ray regime at nanoscales are discussed.

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INTRODUCTION

Recently, there has been a surge in the investigation of collective processes [1–3] in quantum plasmas. The latter are ubiquitous in a variety of physical environments, including the cores of Jupiter [4] and white dwarf stars [5–9], magnetars [10], warm dense matters [11]), compressed plasmas produced by intense laser beams [12–14], and processing devices for modern high-technology (e.g. semiconductors [15], thin films and nano-metallic structures [16]). In quantum plasmas, the electrons are degenerate and obey the Fermi-Dirac distribution function, while non-degenerate strongly correlated ions are coupled with the electrons via the electromagnetic fields. Quantum mechanical effects become important when the inter-electron distance is of the order of the thermal de Broglie wavelength. Here the overlapping of electron wave functions occurs due to Heisenberg's uncertainty and Pauli's exclusion principles [17].

The quantum statistical pressure law for relativistically degenerate electrons was obtained by Chandrasekhar in his classic paper [18]. He found for the electron pressure [18, 19]

$$P_e = \frac{\pi m_e^4 c^5}{3h^3}\left[R(2R^2-3)\sqrt{1+R^2}+3\ln(R+\sqrt{1+R^2})\right], \quad (1)$$

where m_e is the rest mass of the electrons, c the speed of light in vacuum, h the Planck constant, $R=(n_e/n_c)^{1/3}$, n_e the electron number density, and $n_c \simeq 5.9\times10^{29}$ cm^{-3}. The electron pressure P_e can be represented in polytropic form $P_e = P_0(n/n_0)^\gamma$, where we have $P_0 = \pi^{2/3}\hbar^2 n_0^{5/3}/5m_e$ $(3\hbar c n_0^{4/3}/4)$ with $\gamma = 5/3$ $(4/3)$ for non-relativistic (ultra-relativistic) degenerate electrons, and $\hbar = h/2\pi$ is the reduced Planck's constant. The change in the equation of state (from 5/3 to 4/3) due to ultra-relativistic degeneracy of the electrons is responsible for the Chandrasekhar critical mass [18] $M_c \approx (\hbar c/G)^{3/2} m_n^{-2} \approx 1.4 M_\bullet$, where $M_\bullet$ is the solar mass, of a white dwarf star, due to the balance between the gradient of ultra-relativistic electron pressure $(3\hbar c/4) n_e^{4/3}$ and the gravitational force $GM/R_s^2) m_e m_n$, where G is the gravitational constant, M and R_s are the mass and radius of a star, and m_n mass of the nuclei ($n_e m_n = M/R_c^3$).

Besides the quantum statistical electron pressure in dense plasmas, there is also a negative electron pressure P_C due to Coulomb interactions [20], viz.

$$P_C = -\frac{8\pi^3 m_e^4 c^5 R^4}{h^3} \frac{\alpha_0 Z_i^{2/3}}{10\pi^2} \left(\frac{4}{9\pi}\right)^{1/3}, \quad (2)$$

where $\alpha_0 = 1/137$ is the fine structure constant and Z_i the atomic number of ions. Furthermore, there are novel quantum forces associated with i) the quantum electron recoil/electron tunneling effect [21–26] caused by overlapping of electron wave functions due to the Heisenberg uncertainty and Pauli exclusion principles, and ii) electron-exchange and electron correlations [27] caused by electron spin effect [28]. These quantum forces (the quantum pressure gradients, the quantum Bohm force due to the quantum electron recoil effect [21, 22], and the gradient of the the potential associated with electron-exchange and electron-correlations effects [27]), and electromagnetic forces control the dynamics of degenerate electrons in dense quantum plasmas. It turns out that the quantum forces produce new features to both electron and ion plasma oscillations [1-3] in an unmagnetized quantum plasma.

In this paper, we present two novel aspects of collective interactions in quantum plasmas. First, we report on the discovery of novel attractive force [29] at atomic dimensions/nanoscales that can bring ions closer. Novel attractive force arises due to the interplay between the quantum Bohmian force [21, 22], as well as forces due to the quantum statistical pressure [18] of non-relativistic degenerate electrons and electron-exchange and electron correlations effects [27]. Specifically, we shall use here the generalized quantum hydrodynamical (G-QHD) equations [1–3, 29] for non-relativistic degenerate electron fluids, the generalized viscoelastic (GV) ion momentum equations [1] including ion correlations and ion fluid viscosity effects, supplemented by Poisson's equation. Thus, our G-QHD-GV model captures the essential physics of electrostatic collective modes in quantum plasmas.

The manuscript is organized in the following fashion. In Sec. II, we present the governing equations for electrostatic perturbations and derive the electron and ion susceptibilities in an unmagnetized quantum plasma. The linear response theory for quasi-stationary electrostatic perturbations involving immobile ions reveals that the electric potential around an isolated ion in a quantum plasma has a new profile [29], which resembles the Lennard-Jones-type potential. Thus, our newly found electric potential embodies a short-range negative hard core electric potential. The latter would be responsible for ion clustering and oscillations of ion lattices under the new attractive force we have recently discovered [29] in a strongly coupled quantum plasma. Section III contains an investigation of stimulated scattering instabilities of coherent high-frequency electromagnetic (EM) waves off quantum electron and ion plasma modes. Here we discuss stimulated Raman, stimulated Brillouin, and modulational instabilities of a constant amplitude EM pump wave. Explicit expressions for the growth rates are presented. The relevance of our investigation to the field of high-energy density quantum plasma physics is pointed out.

THE GOVERNING EQUATIONS

Let us consider an unmagnetized quantum plasma consisting of non-relativistic degenerate electrons and mildly coupled ions. In our quantum plasma, the electron and ion coupling parameters are $\Gamma_e = e^2/a_e k_B T_F$ and $\Gamma_i = Z_i^2 e^2/a_i k_B T_i$, respectively, where $a_e \sim a_i = (3/4\pi n_0)^{1/3}$ is the average interparticle distance, k_B the Boltzmann constant, $T_F = (\hbar^2/2m_e k_B)(3\pi^2 n_0)^{2/3}$ the Fermi electron temperature, and T_i the ion temperature. It turns out that $\Gamma_i/\Gamma_e = Z_i^2 T_F/T_i \gg 1$, since in quantum plasmas we usually have $T_F > T_i$. The dynamics of degenerate electron fluids is governed by the generalized quantum hydrodynamic (G-QHD) equations composed of the continuity equation

$$\frac{\partial n_e}{\partial t} + \nabla \cdot (n_e \mathbf{u}_e) = 0, \tag{3}$$

the momentum equation

$$m_e n_e \frac{d\mathbf{u}_e}{dt} = n_e e \nabla\phi - \nabla P_* + n_e \nabla V_{xc} + n_e \nabla V_B, \tag{4}$$

and Poisson's equation

$$\nabla^2 \phi = 4\pi e(n_e - n_i) - 4\pi Q \delta(\mathbf{r}), \tag{5}$$

Here $d/dt = (\partial/\partial t) + \mathbf{u}_e \cdot \nabla$, n_e (n_i) is the electron (ion) number density, Q the test charge, $\mathbf{u}_e$ the electron fluid velocity, and ϕ the electrostatic potential. Furthermore, we have denoted the quantum statistical pressure $P_* = (n_0 m_e v_*^2/5)(n/n_0)^{5/3}$, where $v_* = \hbar(3\pi^2)^{1/3}/m_e r_0$ is the electron Fermi speed and $r_0 = n_0^{-1/3}$ represents the mean inter-particle distance. The sum of the electron exchange and electron correlations potential is [27] $V_{xc} = 0.985(e^2/\varepsilon)n^{1/3}[1 + (0.034/a_B n^{1/3})\ln(1 + 18.37 a_B n^{1/3})]$, where $a_B = \hbar^2/m_e e^2$ is the Bohr atomic radius. The quantum Bohm potential reads [1, 3, 21, 22] $V_B = (\hbar^2/2m_e)(1/\sqrt{n_e})\nabla^2\sqrt{n_e}$. Equation (4) is valid if the plasmonic energy density $\hbar\omega_{pe}$ is smaller than or comparable with the Fermi electron kinetic energy $k_B T_F$, where $\omega_{pe} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, and the electron-ion collision relaxation time is greater than the electron plasma period.

The ion number density perturbation n_{i1} is obtained from the ion continuity equation

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \mathbf{u}_i) = 0, \tag{6}$$

where the ion fluid velocity $\mathbf{u}_i$ is determined from the generalized viscoelastic ion momentum equation

$$\left(1+\tau_m\frac{D}{Dt}\right)\left(m_i n_i\frac{D\mathbf{u}_i}{Dt}+\nabla P_i+Z_i e n_i\nabla\phi\right)-\eta\nabla^2\mathbf{u}_i-\left(\xi+\frac{\eta}{3}\right)\nabla(\nabla\cdot\mathbf{u}_i)=0, \quad (7)$$

where $D/Dt=(\partial/\partial t)+\mathbf{u}_i\cdot\nabla$, τ_m is the viscoelastic relaxation time for ion correlations, $P_i=\gamma_i\mu_i k_B T_i\nabla n_i$ the ion thermal pressure involving strong ion coupling effects, m_i the ion mass, γ_i the adiabatic index, $\mu_i[=(1/k_BT_i)(\partial P_i/\partial n_i)_{k_BT_i}]\equiv 1+U_i(\Gamma_i)/3+(\Gamma_i/9)\partial U_i(\Gamma_i)/\partial\Gamma_i$ the isothermal compressibility factor for non-degenerate ion fluids, the function $U_i(\Gamma_i)$ is the measure of the excess internal energy of the system, which is related with the correlation energy E_c by $U_i(\Gamma_i)=E_c/n_ik_BT_i$ [$\equiv\Gamma_i(0.9+1.5r_i^2/a_i^2)$, where r_i is the ion core radius which depends on the degree of ion stripping [30]]. For one-component plasma model, we [31, 32] usually adopt $U_i(\Gamma_i)\approx-0.9\Gamma_i$ for $\Gamma_i\gg 1$ Furthermore, the coefficients of the shear and bulk ion fluid viscosities are denoted by η and ξ, respectively. We note that Eq. (7) is similar to that used by Frenkel [33] and Ichimaru [31, 32] in the context of ordinary fluids and one-component strongly coupled plasmas, respectively. Furthermore, Kaw and Sen [34] adopted a generalized viscoelastic dust momentum equation for studying the properties of dust acoustic waves [35] in a multi-component dusty plasma with highly- charged dust grains.

Letting $n_j=n_0+n_{j1}$, where $n_{j1}\ll n_0$ and j equals e for electrons and i for ions, we linearize Eqs. (3) and (4) as well as (6) and (7) to obtain the electron and ion number density perturbations n_{j1}. The latter can be substituted into the Fourier transformed version of Eq. (5), obtaining, in the linear approximation, the electric potential [29]

$$\phi(\mathbf{r})=\frac{Q}{2\pi^2}\int\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2D(\omega,\mathbf{k})}d^3k, \quad (8)$$

where the dielectric constant $D(\omega,\mathbf{k})$ for an unmagnetized quantum plasma is

$$D(\omega,\mathbf{k})=1+\chi_e(\omega,\mathbf{k})+\chi_i(\omega,\mathbf{k}). \quad (9)$$

Here the electron and ion susceptibilities are, respectively,

$$\chi_e(\omega,\mathbf{k})=-\frac{\omega_{pe}^2}{\omega^2-k^2U_*^2-\hbar^2k^4/4m_e^2}, \quad (10)$$

and

$$\chi_i(\omega,\mathbf{k})=-\frac{\omega_{pi}^2}{\omega^2-k^2V_{ti}^2+i\omega\eta_*k^2/(1-i\omega\tau_m)}, \quad (11)$$

where $U_*=(v_*^2/3+v_{ex}^2)^{1/2}$, $v_{ex}=(0.328e^2/m_er_0)^{1/2}[1+0.62/(1+18.36a_Bn_0^{1/3})]^{1/2}$, $\omega_{pi}=(4\pi n_0Z_i^2e^2/m_i)^{1/2}$, $V_{ti}=(\gamma_i\mu_ik_BT_i/m_i)^{1/2}$ the ion thermal speed, and $\eta_*=(\xi+4\eta/3)/m_in_0$. The frequency and wave vector is denoted by ω and $\mathbf{k}$, respectively.

Linear Quantum Plasma Waves

Two cases are of interest. First, for high-frequency electron plasma waves with $\omega \gg \omega_{pi}$, we have from $1+\chi_e(\omega,\mathbf{k})=0$, the frequency of the electron plasma oscillations (EPOs)

$$\omega(k)=\left(\omega_{pe}^2+k^2U_*^2+\frac{\hbar^2k^4}{4m_e^2}\right)^{1/2}\equiv\Omega_L(k). \tag{12}$$

Second for $\omega \ll k(U_*^2+\hbar^2k^2/4m_e^2)^{1/2}\equiv kU_{q*}$, we have $\chi_e(\mathbf{k})=\omega_{pe}^2/k^2U_{q*}^2$, so that $1+\chi_e+\chi_i=0$ gives

$$\omega^2+i\omega\frac{\omega_v}{(1-i\omega\tau_m)}-k^2C_s^2-\frac{\hbar^2k^4}{4m_em_i}=0, \tag{13}$$

for the ion plasma oscillations (IPOs). Here we have denoted $\omega_v=\eta_*k^2$ and $C_s=(m_eU_*^2/m_i+V_{ti}^2)^{1/2}$. In the hydrodynamic limit, viz. $\omega\tau_m\ll 1$, we have viscous damping of the quantum ion plasma mode. The real and imaginary parts of the frequencies $(\omega=\omega_r+i\omega_i)$ are, respectively,

$$\omega_r(k)=\left[k^2C_s^2+\frac{\hbar^2k^4}{4m_em_i}-\omega_i^2\right]^{1/2}\equiv\Omega_s(k), \tag{14}$$

and

$$\omega_i(k)=-\frac{\omega_v}{2}. \tag{15}$$

Furthermore, in the kinetic regime characterized by $\omega\tau_m\gg 1$, we have from (13)

$$\omega(k)=\left(\frac{\omega_v}{\tau_m}+k^2C_s^2+\frac{\hbar^2k^4}{4m_em_i}\right)^{1/2}\equiv\Omega_I(k). \tag{16}$$

Generally, $\tau_m=\tau_0Y_g(k)$, where $\tau_0=1/\omega_v(1-\mu_i+4U_i(\Gamma_i)/15)$ and $Y_g(k)=\exp(-k/k_g)$ for a Gaussian distribution, and $Y_l(k)=(1+k^2/k_L^2)^{-1}$ for a Lorentzian distribution. Here k_g and k_l are the scale factors [32].

Potential Distribution Around an Isolated Ion

Let us now present novel attractive force [29] in quantum plasmas, by using $1/D$ that is associated with quasi-stationary ($\omega\to 0$)) propagating electrostatic perturbations in a quantum plasma with immobile ions. Accordingly, we have [29]

$$\frac{1}{D}=\frac{(k^2/k_s^2)+\alpha k^4/k_s^4}{1+(k^2/k_s^2)+\alpha k^4/k_s^4}, \tag{17}$$

where $k_s = \omega_{pe}/U_*$ is the inverse modified (due to the potential of the electron-exchange and electron-correlations) Thomas-Fermi screening length, and $\alpha = \hbar^2\omega_{pe}^2/4m_e^2U_*^2$. We note that α depends only on r_0/a_B.

Inserting Eq. (17) into Eq. (8), we can express the latter as [29]

$$\phi(\mathbf{r}) = \frac{Q}{4\pi^2}\int\left[\frac{(1+b)}{k^2+k_+^2}+\frac{(1-b)}{k^2+k_-^2}\right]\exp(i\mathbf{k}\cdot\mathbf{r})d^3k, \tag{18}$$

where $b = 1/\sqrt{1-4\alpha}$, and $k_\pm^2 = k_s^2[1\mp\sqrt{1-4\alpha}]/2\alpha$.

The integral in Eq. (18) can be evaluated using the general formula

$$\int\frac{\exp(i\mathbf{k}\cdot\mathbf{r})}{k^2+k_\pm^2}d^3k = 2\pi^2\frac{\exp(-k_\pm r)}{r}, \tag{19}$$

where the branches of $k_\pm$ must be chosen with positive real parts so that the boundary condition $\phi\to 0$ at $r\to\infty$ is fulfilled. From Eqs. (18) and (19) we then have

$$\phi(\mathbf{r}) = \frac{Q}{r}[\cos(k_i r) - ib\sin(k_i r)]\exp(-k_r r). \tag{20}$$

Several comments are in order. First, for $\alpha > 1/4$, the solution of $k_\pm^2 = k_s^2\left(\mp i\sqrt{4\alpha-1}\right)/2\alpha$ yields $k_\pm = (k_s/\sqrt{4\alpha})[(\sqrt{4\alpha}+1)^{1/2}\mp i(\sqrt{4\alpha}-1)^{1/2}] \equiv k_r \mp ik_i$, and the corresponding potential reads [29]

$$\phi(\mathbf{r}) = \frac{Q}{r}\left[\cos(k_i r) + b_*\sin(k_i r)\right]\exp(-k_r r), \tag{21}$$

where $b_* = 1/\sqrt{4\alpha-1}$. Second, for $\alpha\to 1/4$, we have $k_+ = k_- = \sqrt{2}k_s$ and

$$\phi(\mathbf{r}) = \frac{Q}{r}\left(1+\frac{k_s r}{\sqrt{2}}\right)\exp(-\sqrt{2}k_s r). \tag{22}$$

Third, for $\alpha < 1/4$, the expression $\sqrt{1-4\alpha}$ is real, and we obtain $k_\pm = k_s(1\mp\sqrt{1-4\alpha})^{1/2}/\sqrt{2\alpha}$. The resultant electric potential is

$$\phi(\mathbf{r}) = \frac{Q}{2r}\left[(1+b)\exp(-k_+ r) + (1-b)\exp(-k_- r)\right]. \tag{23}$$

We note that in the absence of the quantum recoil effect, viz. $\alpha\to 0$, we recover from Eq. (23) the modified Thomas-Fermi screened Coulomb potential $\phi(\mathbf{r}) = (Q/r)\exp(-k_s r)$. Finally, in the limit $\hbar^2k^2 \gg m_e^2U_*^2$, the quantum recoil force dominates over the quantum statistical pressure and the forces associated with electron-exchange and electron correlations effects. Here we have the exponential cosine-screened Coulomb potential [36]

$$\phi(\mathbf{r}) = \frac{Q}{r}\cos(k_q r)\exp(-k_q r), \tag{24}$$

where $k_q = (2m_e\omega_{pe}/\hbar)^{1/2}$ is the quantum wave number.

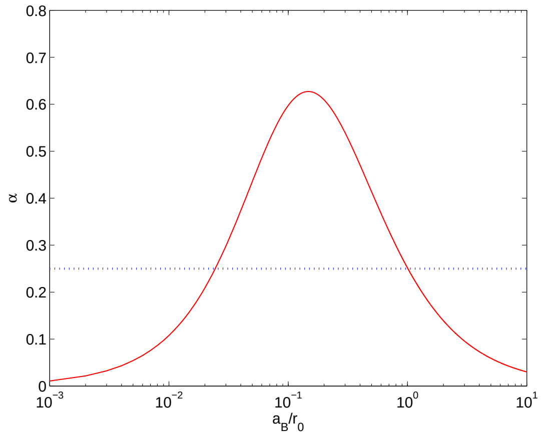

FIGURE 1. The variation of α against a_B/r_0. The critical value $\alpha = 1/4$ is indicated with a dotted line.

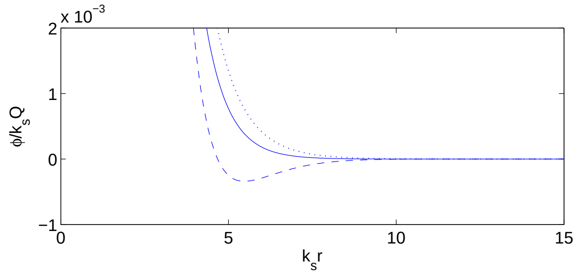

FIGURE 2. The electric potential ϕ as a function of r for $\alpha = 0.625$ (dashed curve), $\alpha = 1/4$ (solid curve) and $\alpha = 0$ (dotted curve). The value 0.627 is the maximum possible value of α in our model, obtained for $a_B/r_0 \approx 0.15$. After P. K. Shukla and B. Eliasson, *Phys. Rev. Lett.* **108**, 219902 (2012).

Figure 1 displays the variation of α against a_B/r_0, with a maximum α at $a_B/r_0 \approx 0.15$, corresponding to a number density $n_0 \approx 2\times 10^{22}\,\mathrm{cm}^{-3}$ (with $a_B = 5.3\times 10^{-9}$ cm), a few times below solid densities. The value of α is above the critical value $1/4$ only for a limited range of the plasma densities. In Fig. 2, we depict the profiles of the potential [given by Eqs. (21), (22) and (23) for $\alpha > 1/4$, $\alpha = 1/4$ and $\alpha < 1/4$, respectively] for different values of α. We clearly see the new short-range attractive electric potential that resembles the LJ-type potential for $0.25 < \alpha < 0.627$, while for $\alpha \leq 0.25$, the attractive potential is absent. Figure 3(a) exhibits the distance $r = d$ from the test ion charge where $d\phi/dr = 0$ and the electric potential has its minimum, and Figs. 3(b) and 3(c) show the values of ϕ and $d^2\phi/dr^2$ at $r = d$. The value of $(d^2\phi/dr^2)$ determines the frequency and dispersion properties of the ion plasma oscillations, as shown below.

The interaction potential energy between two dressed ions with charges Q_i and Q_j at the positions $\mathbf{r}_i$ and $\mathbf{r}_j$ can be represented as $U_{i,j}(\mathbf{R}_{ij}) = Q_j\phi_i(\mathbf{R}_{ij})$, where ϕ_i is the potential around particle i, and $\mathbf{R}_{ij} = \mathbf{r}_i - \mathbf{r}_j$. For $\alpha > 1/4$, it reads, using Eq. (21),

$$U_{i,j}(\mathbf{R}_{ij}) = \frac{Q_iQ_j}{|\mathbf{R}_{ij}|}\exp(-k_r|\mathbf{R}_{ij}|)\left[\cos(k_i|\mathbf{R}_{ij}|) + b_*\sin(k_i|\mathbf{R}_{ij}|)\right]. \tag{25}$$

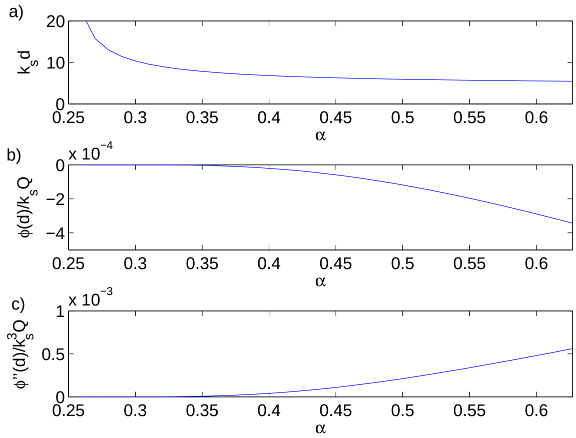

FIGURE 3. a) The distance $r = d$ from the test ion charge where $d\phi/dr = 0$ and the electric potential has its minimum, and b) the values of the potential ϕ and c) its second derivative $d^2\phi/dr^2$ at $r = d$. After P. K. Shukla and B. Eliasson, *Phys. Rev. Lett.* **108**, 219902 (2012).

On account of the interaction potential energy, ions would suffer vertical oscillations around their equilibrium position. The vertical vibrations of ions in a crystallized ion string in quantum plasmas are governed by

$$m_i \frac{d^2 \delta z_j(t)}{dt^2} = -\sum_{i \neq j} \frac{\partial U_{ij}(\mathbf{r}_i, \mathbf{r}_j)}{\partial z_j}, \tag{26}$$

where $\delta z_j(t)[= z_j(t) - z_{j0}]$ is the vertical displacement of the jth ion from its equilibrium position z_{j0}. Assuming that $\delta z_j(t)$ is proportional to $\exp[-i(\omega t - jka)]$, where ω and k are the frequency and wave number of the ion lattice oscillations, respectively, and that $Q_i = Q_j = Q$, Eq. (26) for the nearest-neighbor ion interactions gives

$$\omega^2 = \frac{4Q^2}{m_i d^3} S \exp(-k_r d) \sin^2\left(\frac{kd}{2}\right), \tag{27}$$

where $S = [2(1 + k_r d) + (k_r^2 - k_i^2)d^2](\cos\xi + b_* \sin\xi) + 2k_i d(1 + k_r d)(\sin\xi - b_* \cos\xi)$. $\xi = k_i d$, and d is the separation between two consecutive ions. We note that Eq. (27) can also be obtained from the formula [37]

$$\omega^2 = \frac{4}{m_i}\left[\frac{d^2 W(r)}{dr^2}\right]_{r=d} \sin^2\left(\frac{kd}{2}\right), \tag{28}$$

where the inter-ion potential energy for $\alpha > 1/4$ is represented as

$$W(r) = (Q^2/r)\exp(-k_r r)\left[\cos(k_i r) + b_* \sin(k_i r)\right].$$

Hence, the lattice wave frequency is proportional to $[d^2\phi(r)/dr^2]_{r=d}^{1/2}$ [cf. Fig. 3(c)].

STIMULATED SCATTERING OF ELECTROMAGNETIC WAVES

Large amplitude high-frequency (HF) electromagnetic (EM) waves are used for heating inertial confined fusion plasmas [38, 39], as well as for diagnostic purposes [11] in solid density plasmas that are created by intense laser and charged particle beams. The HF EM pulses also appear as localized bursts of x-rays and γ-rays from compact astrophysical objects. Furthermore, the generation of coherent HF EM waves is of great importance in the context of free-electron lasers (FELs) involving EM wigglers [40, 41, 42]. Therefore, studies of nonlinear phenomena (e.g parametric instabilities [43] and HF EM wave localizations [44]) associated with a large amplitude HF EM waves in dense quantum plasmas are of practical interest.

In the following, we present an investigation of stimulated scattering instabilities of coherent circular polarized electromagnetic (CPEM) waves carrying orbital angular momentum (OAM) in an unmagnetized dense quantum plasma. The nonlinear interactions between the CPEM waves and electrostatic plasma oscillations in quantum plasmas is governed by the EM wave equation [42, 45]

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2 + \omega_p^2\right)\mathbf{A} + \omega_{pe}^2 N\mathbf{A} = 0, \tag{29}$$

which is derived from the Maxwell equation and the electron equation of motion with the EM fields $\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t$ and $\mathbf{B} = \nabla\times\mathbf{A}$, using the Coulomb gauge $\nabla\cdot\mathbf{A} = 0$. Here $\mathbf{A}$ is the vector potential, $N = n_{e1}/n_0 \ll 1$, and n_{e1} the electron number density perturbation associated with low-frequency electrostatic plasma oscillations (EPOs) that are reinforced by the ponderomotive force of the CPEM waves. In the absence of nonlinear couplings between the latter and the EPOs, the paraxial EM wave solution, $\mathbf{A}(r,z)\exp(-i\omega_e t + ik_e z)$, where $\omega_e = (k_e^2 c^2 + \omega_{pe}^2)^{1/2}$ and k_e are the frequency and the wave number of the CPEM wave, respectively, of Eq. (28) is [46, 47]

$$\mathbf{A}(r,z) = \mathbf{A}F_{p,l}(r,z)\exp(il\varphi). \tag{30}$$

Here we have denoted

$$F_{p,l}(r,z) = \frac{1}{2\sqrt{\pi}}\left[\frac{(l+p)!}{p!}\right]^{1/2} X^{|l|}L_p^{|l|}(X)\exp(-X/2), \tag{31}$$

where $X = r^2/w^2(z)$, $w(z)$ the beam waist, and the associated Laguerre polynomial $L_p^{|l|}(X)$ are defined by the Rodriguez formula, $L_p^{|l|}(X) = (X^l p!)^{-1}\exp(X)$ $d^p[X^{(l+p)}\exp(-X)]/dX^p$, p and l are the radial and angular mode numbers of the EM orbital angular momentum states, respectively, φ the azimuthal angle, $r = (x^2+y^2)^{1/2}$ the radial of the cylindrical coordinates $((r,\varphi,z)$, so that $\nabla^2 = \nabla_\perp^2 + \partial_z^2$, where $\nabla_\perp^2 = (1/r)(\partial/\partial r)(r\partial/\partial r) + (1/r^2)\partial^2/\partial\varphi^2$. The Laguerre-Gauss (LG) solutions (30) describe CPEM waves with a finite OAM.

The dynamics of the low-frequency (in comparison with the CPEM wave frequency ω_e) plasma oscillations involving degenerate electrons and non-degenerate ion fluids is governed by the G-QHD equations composed of the linearized electron continuity equation

$$\frac{\partial n_{e1}}{\partial t} + n_0 \nabla \cdot \mathbf{u}_e = 0, \tag{32}$$

the electron momentum equation

$$\frac{\partial \mathbf{u}_e}{\partial t} - \frac{e}{m_e}\nabla(\phi - \phi_p) + U_*^2 \nabla n_{e1} + \frac{\hbar^2}{4m_e^2}\nabla\nabla^2 n_{e1} = 0, \tag{33}$$

and Poisson's equation

$$\nabla^2 \phi = 4\pi e(n_{e1} - n_{i1}), \tag{34}$$

together with the linearized Eqs. (6) and (7). Here $\phi_p = e|\mathbf{A}|^2/m_e c^2$ is the ponderomotive potential of the CPEM waves. The EM ponderomotive force [38, 39] $-e\nabla\phi_p$ comes from the averaging (over the CPEM wave period) of the advection and nonlinear Lorentz forces involving the electron quiver velocity and the laser wave magnetic field.

Let us now derive the governing equations for the electron and ion plasma oscillations in the presence of the ponderomotive force of the CPEM waves. First, we consider the driven electron plasma oscillations on the time scale of the electron plasma period, so that the ions do not have time to respond and the ion density perturbation is zero. We combine Eqs. (32) and (33) and substitute the resultant equation into (34) with $n_{i1} = 0$ to obtain the electron plasma wave equation [45]

$$\left(\frac{\partial^2}{\partial t^2} + \omega_{pe}^2 - U_*^2\nabla^2 + \frac{\hbar^2}{4m_e^2}\nabla^4\right) N = \frac{e^2}{2m_e^2 c^2}\nabla^2|\mathbf{A}|^2. \tag{35}$$

Second, we consider driven ion oscillations by supposing that $|\partial^2 N/\partial t^2| \ll |U_*^2\nabla^2 N + (\hbar^2/4m_e^2)\nabla^4 N|$. Here, one can neglect the right-hand side in Eq. (33), and use the resultant equation to eliminate the electric field $-\nabla\phi$ from Eq. (7) to obtain, after linearization of the resultant equation under the quasi-neutral approximation $n_{e1} = n_{i1}$, the driven ion oscillation equation [45]

$$\left(1 + \tau_m \frac{\partial}{\partial t}\right)\left(\frac{\partial^2}{\partial t^2} - C_s^2\nabla^2 + \frac{\hbar^2}{4m_e m_i}\nabla^4\right) N$$
$$- \frac{\eta}{m_i n_0}\nabla\cdot\nabla\frac{\partial N}{\partial t} - \frac{\left(\xi + \frac{\eta}{3}\right)}{m_i n_0}\nabla^2\frac{\partial N}{\partial t} = \left(1 + \tau_m\frac{\partial}{\partial t}\right)\frac{Z_i^2 e^2}{2m_0 m_i c^2}\nabla^2|\mathbf{A}|^2, \tag{36}$$

where we have used the linearized Eq. (6) to eliminate $\nabla\cdot\mathbf{u}_i$ from the ion momentum equation (7).

Equations (29), (35) and (36) are the desired equations [45] for studying nonlinear effects (viz. parametric instabilities [38] and localization of light pulses [39] associated with L-G CPEM beams in quantum plasmas at nanoscales.

Nonlinear Dispersion Relations

In the following, we present an investigation of stimulated Raman, stimulated Brillouin, and modulational instabilities [48] of L-G CPEM waves. Accordingly, we decompose the vector potential as

$$\mathbf{A} = \mathbf{A}_{0+}\exp(-i\omega_0 t + i\mathbf{k}_0\cdot\mathbf{r}) + \mathbf{A}_{0-}\exp(i\omega_0 t - i\mathbf{k}_0\cdot\mathbf{r}) + \sum_{+,-}\mathbf{A}_{\pm}\exp(-i\omega_{\pm}t + i\mathbf{k}_{\pm}\cdot\mathbf{r}), \tag{37}$$

where the subscripts 0 and $\pm$ denote the CPEM pump and CPEM sidebands, respectively, and $\omega_{\pm} = \Omega \pm \omega_0$ and $\mathbf{k}_{\pm} = \mathbf{K} \pm \mathbf{k}_0$ are the frequency and wave vectors of the CPEM sidebands that are created by the beating of the pump $(\omega_0, \mathbf{k}_0)$ and electrostatic oscillations $(\Omega, \mathbf{K})$.

Inserting (37) into (29), (35), and (36), and supposing that N is proportional to $\exp(-i\Omega t + i\mathbf{K}\cdot\mathbf{r})$, we Fourier decompose the resultant equations to obtain the nonlinear dispersion relations [44]

$$S_R = \frac{\omega_{pe}^2 e^2 K^2}{2m_e^2 c^2}\sum_{+,-}\frac{|\mathbf{A}_0 F_{p,l}|^2}{D_{\pm}}, \tag{38}$$

and

$$S_B = \frac{\omega_{pe}^2 Z_i^2 e^2 K^2}{2m_e m_i c^2}\sum_{+,-}\frac{|\mathbf{A}_0 F_{p,l}|^2}{D_{\pm}}. \tag{39}$$

Here we have denoted $S_R = \Omega^2 - \omega_{pe}^2 - K^2 U_*^2 - \hbar^2 K^4/4m_e^2$ and $S_B = \Omega^2 - K^2 C_s^2 - (\hbar^2 K^4/4m_e m_i) + i\Omega\eta_* K^2/m_i n_0(1 - i\Omega\tau_m)$, and $D_{\pm} = \omega_{\pm}^2 - k_{\pm}^2 c^2 - \omega_{pe}^2 \approx \pm 2\omega_0(\Omega - \mathbf{K}\cdot\mathbf{V}_g \mp \delta)$, where $\mathbf{V}_g = c^2\mathbf{k}_0/\omega_0$ is the group velocity of the CPEM pump, $\omega_0 = (\omega_{pe}^2 + k_0^2 c^2)^{1/2}$ the pump wave frequency, and $\delta = K^2 c^2/2\omega_0$ the small frequency shift arising from the nonlinear interaction between the CPEM pump and the electrostatic plasma oscillations in our quantum plasma.

Growth Rates

We now present a summary of formulas for the growth rates of stimulated Raman (SR) and stimulated Brillouin (SB) scattering instabilities, as well as of modulational instabilities of a constant amplitude pump that is scattered off a quantum electron plasma wave, a quantum ion mode, and a spectrum of non-resonant electron and ion density perturbations. For three-wave decay interactions, one assumes that $D_- = 0$ and $D_+ \neq 0$. Thus, one ignores D_+ from Eqs. (38) and (39). Letting $\Omega = \mathbf{K}\cdot\mathbf{V}_g - \delta + i\gamma_{R,B}$, and $\Omega = \Omega_L + i\gamma_R$ and $\Omega = \Omega_s\,(\Omega_I) + i\gamma_B$ in the resultant equations, we obtain the growth rates for Raman and Brillouin backscattering ($|\mathbf{K}| = 2k_0$) instabilities, respectively,

$$\gamma_R = \frac{\omega_{pe} k_0 e |\mathbf{A}_0 F_{p,l}|}{\sqrt{2\omega_e \Omega_R} m_e c}, \tag{40}$$

and

$$\gamma_B = \frac{\omega_{pe} k_0 Z_i e |\mathbf{A}_0 F_{p,l}|}{\sqrt{2\omega_0 \Omega_B m_e m_i} c}, \tag{41}$$

where $\Omega_R = \Omega_L(K = 2k_0)$, $\Omega_B = \Omega_{s,I}(K = 2k_0)$, and $|2k_0 \mathbf{V_g} - \delta| \sim \Omega_R, \Omega_B$.

Since the growth rates of SR and SB scattering instabilities, given by Eqs. (40) and (41), respectively, are proportional to Ω_R and Ω_B, one notices that quantum and strong ion correlation effects significantly affect the e-folding time of the instabilities. Furthermore, the growth rates, which are proportional to $F_{p,l}$, are minimum at the center of the vortex pump wave with OAM.

Next, for the modulational instabilities, we have $D_\pm \neq 0$ and $S_{R,B} \neq 0$. Here, we have to retain both upper and lower CPEM sidebands on an equal footing in (38) and (39), and write them as

$$S_R \left[(\Omega - \mathbf{K} \cdot \mathbf{V}_g)^2 - \delta^2\right] = \frac{\delta \omega_{pe}^2 e^2 K^2 |\mathbf{A}_0 F_{p,l}|^2}{2\omega_0 m_e^2 c^2}, \tag{42}$$

and

$$S_B \left[(\Omega - \mathbf{K} \cdot \mathbf{V}_g)^2 - \delta^2\right] = \frac{\delta \omega_{pe}^2 Z_i^2 e^2 K^2 |\mathbf{A}_0 F_{p,l}|^2}{2\omega_0 m_e m_i c^2}. \tag{43}$$

Equations (42) and (43) can be analyzed numerically to obtain the growth rates of the modulational instabilities. However, some analytical results follow for $\mathbf{K} \cdot \mathbf{V}_g = 0$, in which case we have from (42) and (43), respectively,

$$\Omega^2 = \frac{1}{2}(\Omega_L^2 + \delta^2) \pm \frac{1}{2}\left[(\Omega_L^2 - \delta^2)^2 + \frac{2\delta \omega_{pe}^2 e^2 K^2 |\mathbf{A}_0 F_{p,l}|^2}{\omega_0 m_e^2 c^2}\right]^{1/2}, \tag{44}$$

and

$$\Omega^2 = \frac{1}{2}(\Omega_{s,I}^2 + \delta^2) \pm \frac{1}{2}\left[(\Omega_{s,I}^2 - \delta^2)^2 + \frac{2\delta \omega_{pe}^2 Z_i^2 e^2 K^2 |\mathbf{A}_0 F_{p,l}|^2}{\omega_0 m_e m_i c^2}\right]^{1/2}, \tag{45}$$

which exhibit oscillatory modulational instabilities.

SUMMARY AND CONCLUSIONS

In this paper, we have discussed two new aspects of collective interactions in an unmagnetized quantum plasma. First, we have reported the existence of novel attractive force that can bring ions closer. The appearance of the novel attractive force is attributed to tunneling of degenerate electrons through the Bohmian potential on account of overlapping electron wave functions at atomic dimensions. There are several consequences of our newly found short-range attractive force at quantum scales. For example, due to the trapping of ions in the negative part of the exponential oscillating-screened Coulomb potential, there will arise ordered ion structures/ion clustering depending on the electron density concentration, which in fact controls the Wigner-Seitz radius r_0. The formation of ion clusters/ion atoms will emerge as new features in a dense quantum plasma. Furthermore, we have shown that both the electron and ion plasma oscillations in quantum plasmas can be excited by a large amplitude CPEM waves due to stimulated Raman and Brillouin scattering instabilities. We also have the possibility of the modulational instabilities of the CPEM waves, via which non-resonant electron density perturbations are created. Hence, there are enhanced electrostatic fluctuations at nanoscales in dense quantum plasmas. In conclusion, we stress that the results of the present investigation are useful for understanding the novel phenomena of ion clustering and the excitation of density fluctuations by the CPEM waves at nanoscales in high-energy density plasmas that are of interest for inertial confinement fusion schemes. Specifically, stimulated scattering instabilities of a coherent high-frequency short-wavelength radiation can offer a possible mechanism for the quantum free-electron lasers in the x-ray and gamma-ray regimes [42[, in addition to serving a diagnostic tool for determining the plasma parameters (viz, the plasma number density), once the spectra of enhanced density fluctuations are experimentally measured by using x-ray spectroscopic techniques. Finally, the present investigation, which has revealed the new physics of collective interactions between an ensemble of electrons at nanoscales, will open a new window for research in one of the modern areas of physics dealing with strongly correlated degenerate electrons and non-degenerate mildly coupled ions in dense quantum plasmas that share knowledge with cooperative phenomena (e.g. the formation of ion lattices) in condensed matter physics and in astrophysics.

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