GENERATING THE MASS OF PARTICLES FROM EXTENDED THEORIES OF GRAVITY

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Abstract. A geometrical approach to produce the mass of particles is derived. The results could be suitably tested at LHC. Starting from a 5D unification scheme, we show that all the known interactions could be induced by a symmetry breaking of the non-unitary $GL(4)$-group of diffeomorphisms. The further gravitational degrees of freedom, emerging from the reduction mechanism in 4D, eliminate the hierarchy problem generating a cut-off comparable with electroweak scales.

1. Introduction

The *Standard Model of Particles* can be considered a successful relativistic quantum field theory both from particle physics and group theory points of view. Technically, it is a non-Abelian gauge theory (a Yang-Mills theory) associated to the tensor product of the internal symmetry groups $SU(3) \times SU(2) \times U(1)$, where the $SU(3)$ color symmetry for quantum chromodynamics is treated as exact, whereas the $SU(2) \times U(1)$ symmetry, responsible for the electro-weak gauge fields, is considered spontaneously broken.

So far, as we know, there are four fundamental forces in Nature; namely, electromagnetic, weak, strong and gravitational forces. The Standard Model well represents the first three, but not the gravitational interaction. On the other hand, General Relativity (GR) is a geometric theory of the gravitational field which is described by the metric tensor $g_{\mu\nu}$ defined on pseudo-Riemannian space-times. The Einstein field equations are nonlinear and have to be satisfied by the metric tensor. This nonlinearity is indeed a source of difficulty in quantization of GR. Since the Standard Model is a gauge theory where all the fields mediating the interactions are represented by gauge potentials, the question is why the fields mediating the gravitational interaction are different from those of the other fundamental forces. It is reasonable to expect that there may be a gauge theory in which the gravitational fields stand on the same footing as those of the other fields. As it is well-known, this expectation has prompted a re-examination of GR from the point of view of gauge theories. While the gauge groups involved in the Standard Model are all internal symmetry groups, the gauge groups in GR is associated to external space-time symmetries. Therefore, the gauge theory of gravity cannot be dealt under the standard of the usual Yang-Mills theories.

Nevertheless, the idea of an unification theory, capable of describing all the fundamental interactions of physics under the same standard, has been one of the main issues of modern physics, starting from the early efforts by Einstein, Weyl, Kaluza and Klein until the most recent approaches. In any case, the large number of ideas, up to now proposed, results unsuccessful due to several reasons: the technical difficulties connected with the lack of a unitary mathematical description of all the interactions; the huge number of parameters introduced to "build up" the unified theory and the fact that most of them cannot be observed neither at laboratory nor at astrophysical (or cosmological) scales; the very wide (and several times questionable since not-testable) number of extra-dimensions requested by several approaches. Due to this situation, it seems that unification is a useful (and aesthetic) paradigm, but far to be achieved, if the trend is continuing to unify interactions by adding and adding new particles and new parameters (e.g. dark matter forest).

A different approach could be to consider the very essential physical quantities and try to achieve unification without any *ad hoc* new ingredients. This approach can be pursued starting from straightforward considerations which lead to reconsider modern physics under a sort of economic issue aimed to unify forces without adding new parameters. A prominent role in this view deserves conservation laws and symmetries.

As a general remark, the Noether Theorem states that, for every conservation law of Nature, a symmetry *must* exist. This leads to a fundamental result also from a mathematical point of view since the presence of symmetries technically reduces dynamics (*i.e.* gives rise to first integrals of motion) and, in several cases, allows to get the general solution. With these considerations in mind, we can try to change our point of view and investigate what will be the consequences of the absolute validity of conservation laws without introducing any arbitrary symmetry breaking [1].

In order to see what happens as soon as we ask for the absolute validity of conservation laws, we could take into account the Bianchi identities. Such geometrical identities work in every covariant field theory (*e.g.* Electromagnetism or GR) and can be read as equations of motion also in a fiber bundle approach [2]. It is possible to show that, the absolute validity of conservation laws, intrinsically contains symmetric dynamics; moreover, reducing dynamics from 5D to 4D, it gives rise to the physical quantities characterizing particles as the mass [3].

The *minimal* ingredient which we require is the fact that a 5-dimensional, singularity free space, where conservation laws are always and absolutely *conserved*, has to be defined. Specifically, in such a space, Bianchi identities are asked to be always valid and, moreover, the process of reduction to 4D-space *generates* the mass spectra of particles. In this sense, a dynamical unification scheme will be achieved where a fifth dimension has the physical meaning of *inducing the mass of particles*. In other words, "effective" scalar fields coming from dimensional reduction mechanisms are related to the $GL(4)$-group of diffeomorphisms. In this sense, we do not need any spontaneous symmetry breaking but just a self-consistent way to classify space-time deformations and reductions as "gauge bosons" [3].

2. The 5D-space and the reduction to 4D-dynamics

Let us start with a 5D-variational principle with

$$\delta \int d^{(5)}x\sqrt{-g^{(5)}}\left[{}^{(5)}R + \lambda(g_{44} - \varepsilon\Phi^2)\right] = 0\,, \tag{1}$$

where λ is a Lagrange multiplier, Φ a scalar field and $\varepsilon = \pm 1$. This approach is completely general and used in theoretical physics when we want to put in evidence some specific feature [4]. In this case, we need it in order to derive the physical gauge for the 5D-metric. We can write the metric as

$$dS^2 = g_{AB}dx^A dx^B = g_{\alpha\beta}dx^\alpha dx^\beta + g_{44}(dx^4)^2 = g_{\alpha\beta}dx^\alpha dx^\beta + \varepsilon\Phi^2(dx^4)^2\,, \tag{2}$$

from which we obtain directly particle-like solutions ($\varepsilon = -1$) or wave-like solutions ($\varepsilon = +1$) in the 4D-reduction procedure. The standard signature of 4D-component of the metric is $(+---)$ and $\alpha, \beta = 0, 1, 2, 3$. Furthermore, the 5D-metric can be written in a Kaluza-Klein fashion as the matrix

$$g_{AB} = \begin{pmatrix} g_{\alpha\beta} & 0 \\ 0 & \varepsilon\Phi^2 \end{pmatrix}\,, \tag{3}$$

and the 5D-curvature Ricci tensor is

$${}^{(5)}R_{\alpha\beta} = R_{\alpha\beta} - \frac{\Phi_{,\alpha;\beta}}{\Phi} + \frac{\varepsilon}{2\Phi^2}\left(\frac{\Phi_{,4}g_{\alpha\beta,4}}{\Phi} - g_{\alpha\beta,44} + g^{\lambda\mu}g_{\alpha\lambda,4}g_{\beta\mu,4} - \frac{g^{\mu\nu}g_{\mu\nu,4}g_{\alpha\beta,4}}{2}\right)\,, \tag{4}$$

where $R_{\alpha\beta}$ is the 4D-Ricci tensor. The expressions for ${}^{(5)}R_{44}$ and ${}^{(5)}R_{4\alpha}$ can be analogously derived. After the projection from 5D to 4D, $g_{\alpha\beta}$, derived from g_{AB}, no longer explicitly depends on x^4. From Eq.(4), a useful expression for the Ricci scalar can be derived:

$${}^{(5)}R = R - \frac{1}{\Phi}\Box\Phi\,, \tag{5}$$

where the dependence on ε is explicitly disappeared and $\Box$ is the 4D-d'Alembert operator. The action in Eq.(1) can be recast in a 4D-reduced Brans-Dicke form

$$\mathcal{A} = -\frac{1}{16\pi G_N}\int d^4x\sqrt{-g}\,[\Phi R + \mathcal{L}_\Phi]\,, \tag{6}$$

where the Newton constant is given by

$$G_N = \frac{{}^{(5)}G}{2\pi l} \tag{7}$$

with l a characteristic length in 5D which can be related to a suitable Compton length. Defining a generic function of a 4D-scalar field ϕ as

$$-\frac{\Phi}{16\pi G_N} = F(\phi)\,, \tag{8}$$

we get, in 4D, a general action in which gravity is non-minimally coupled to a scalar field, that is

$$\mathcal{A} = \int_{\mathcal{M}} d^4x\sqrt{-g}\left[F(\phi)R + \frac{1}{2}g^{\mu\nu}\phi_{;\mu}\phi_{;\nu} - V(\phi)\right] + \int_{\partial\mathcal{M}} d^3x\sqrt{-b}K\,, \tag{9}$$

where the form and the role of $V(\phi)$ are still general. The second integral is a boundary term where $K \equiv h^{ij}K_{ij}$ is the trace of the extrinsic curvature tensor K_{ij} of the hypersurface $\partial\mathcal{M}$ which is embedded in the 4D-manifold $\mathcal{M}$; b is the metric determinant of the 3D-manifold. The Einstein field equations can be derived by varying with respect to the 4D-metric $g_{\mu\nu}$

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \tilde{T}_{\mu\nu}\,, \tag{10}$$

where

$$\tilde{T}_{\mu\nu} = \frac{1}{F(\phi)}\left\{-\frac{1}{2}\phi_{;\mu}\phi_{;\nu} + \frac{1}{4}g_{\mu\nu}\phi_{;\alpha}\phi^{;\alpha} - \frac{1}{2}g_{\mu\nu}V(\phi) - g_{\mu\nu}\Box F(\phi) + F(\phi)_{;\mu\nu}\right\} \tag{11}$$

is the effective stress–energy tensor containing the non-minimal coupling contributions, the kinetic terms and the potential of the scalar field ϕ. In the case in which $F(\phi)$ is a constant F_0 (in our units, $F_0 = -1/(16\pi G_N)$), we get the stress–energy tensor of a scalar field minimally coupled to gravity, that is

$$T_{\mu\nu} = \phi_{;\mu}\phi_{;\nu} - \frac{1}{2}g_{\mu\nu}\phi_{;\alpha}\phi^{;\alpha} + g_{\mu\nu}V(\phi)\,. \tag{12}$$

By varying with respect to ϕ, we get the 4D-Klein–Gordon equation

$$\Box\phi - RF'(\phi) + V'(\phi) = 0, \tag{13}$$

where $F'(\phi) = dF(\phi)/d\phi$ and $V'(\phi) = dV(\phi)/d\phi$. It is possible to show that Eq.(13) is nothing else but the contracted Bianchi identity. This feature shows that the effective stress–energy tensor at right hand side of (10) is a zero–divergence tensor and this fact is fully compatible with Einstein theory of gravity also if we started from a 5D-space. Specifically, the reduction procedure, which we have used, preserves the standard features of GR since we are in the realm of the conformal-affine structure [3].

In order to give a physical meaning to the fifth dimension, let us recast the above Klein-Gordon Eq. (13) as

$$\left(\Box + m_{eff}^2\right)\phi = 0\,, \tag{14}$$

where

$$m_{eff}^2 = \left[V'(\phi) - RF'(\phi)\right]\phi^{-1}\,, \tag{15}$$

is the effective mass, *i.e.* a function of ϕ, where self-gravity contributions, $RF'(\phi)$, and scalar field self-interactions, $V'(\phi)$, are taken into account. In any quantum field theory formulated on curved space-times, these contributions, at one-loop level, have the same "weight" [5]. This toy model shows that a "natural" way to generate particle masses can be achieved starting from a 5D picture. In other words, the concept of *mass* can be derived from a very geometrical viewpoint.

3. Massive Gravitational States and the induced symmetry breaking

The above results could be interesting to investigate quantum gravity effects and symmetry breaking in the range between GeV and TeV scales. Such scales are actually investigated by the today running experiments at LHC. It is important to stress that any ultra-violet model of gravity (e.g. at TeV scales) have to explain also the observed weakness of gravitational effects at largest (infra-red) scales. This means that massless (or quasi-massless) modes have to be considered in any case.

The above 5D-action is an example of higher dimensional action where the effective gravitational energy scale (Planck scale) can be "rescaled" according to Eqs. (7) and (8). In terms of mass, being $M_p^2 = \frac{c\hbar}{G_N}$ the constraint coming from the ultra-violet limit of the theory (10^{19} GeV), we can set $M_p^2 = M_\sharp^{D-2}V_{D-4}$, where V_{D-4} is the "volume" coming from the extra dimension. It is easy to see that V_{D-4}, in the 5D case, is related to the fifth component of Φ. $M_\sharp$ is a cut-off mass that becomes relevant as soon as the Lorentz invariance is violated. Such a scale could be of TeV order. As we have shown, it is quite natural to obtain effective theories containing scalar fields of gravitational origin. In this sense, $M_\sharp$ is the result of a dimensional reduction. To be more explicit, the 4D dynamics is led by the effective potential $V(\phi)$ and the non-minimal coupling $F(\phi)$. Such functions could be experimentally tested since related to massive states. In particular, the effective model, produced by the reduction mechanism from 5D to 4D, can be chosen as

$$\mathcal{A} = \int d^4x\,\sqrt{-g}\,\left[-\frac{\phi^2}{2}R\; + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V\right] \tag{16}$$

plus contributions of ordinary matter terms. The potential for ϕ can be assumed as

$$V(\phi) = \frac{M_\sharp^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4\,, \tag{17}$$

where massive and self-interaction terms are present. This is the standard choice of quantum field theory which perfectly fits with the arguments of dimensional reduction. Let us recall again that the scalar field ϕ is not put *by* *hand* into dynamics but it is given by the extra degrees of freedom of gravitational field generated by the reduction process in 4D. It is easy to derive the vacuum expectation value of ϕ, being

$$M_\sharp^2 = 2\lambda M_p^2 \,, \tag{18}$$

which is a fundamental scale of the theory. Such a scale can be confronted with the Higgs vacuum expectation value which is 246 GeV and then with the *hierarchy problem.* If $M_\sharp$ is larger than the Higgs mass, the problem is obviously circumvented. It is important to recall that hierarchy problem occurs when couplings and masses of effective theories are very different than the parameters measured by experiments. This happen since measured parameters are related to the fundamental parameters by renormalization and fine cancellations between fundamental quantities and quantum corrections are necessary. The hierarchy problem is essentially a fine-tuning problem. In particle physics, the question is why the weak force is stronger and stronger than gravity. Both of these forces involve constants of Nature: Fermi's constant for the weak force and Newton's constant for gravity. From the Standard Model, it appears that Fermi's constant is unnaturally large and should be closer to Newton's constant.

Technically, the question is why the Higgs boson is so much lighter than the Planck mass (or the grand unification energy). In fact, researchers are searching for Higgs masses ranging from 115 up to 350 GeV with different selected decay channels from $b\bar{b}$ to $t\bar{t}$ (see for example [6] and refereces therein). One would expect that the large quantum contributions to the square of the Higgs boson mass would inevitably make the mass huge, comparable to the scale at which new physics appears, unless there is an incredible fine-tuning cancellation between the quadratic radiative corrections and the bare mass. With this state of art, the problem cannot be formulated in the context of the Standard Model where the Higgs mass cannot be calculated. In a sense, the problem is solvable if, in a given effective theory of particles, where the Higgs boson mass is calculable, there are no fine-tunings. If one accepts the *big-desert* assumption and the existence of a hierarchy problem, some new mechanism (at Higgs scale) becomes necessary to avoid fine-tunings.

The model which we are discussing contains a "running" scale that could avoid to set precisely the Higgs scale. If the mass of the field ϕ is in TeV region, there is no hierarchy problem being ϕ a gravitational scale. In this case, the Standard Model holds up plus an extended gravitational sector derived from the fifth dimension.

In other words, the Planck scale can be dynamically derived from the vacuum expectation value of ϕ. In some sense, our model, in its low energy realization, works like the model proposed by Antoniadis et al [7]. The Planck scale can be recovered, as soon as the coupling λ is of the order 10^{-31}. Action (16) is an effective model valid up to a cutoff scale of a few $M_\sharp \sim$ TeV (see also [8]). The tiny value of λ, coming from the extra dimension, allows the presence of physical (quasi-) massless gravitons with very large interaction lengths [9].

Also the string theory limit corresponds to a large scalar field vacuum expectation value at TeV [7]. It is important to stress that, by a conformal transformation from the Jordan frame to the Einstein frame, the Planck scale is decoupled from the vacuum expectation of the scalar field ϕ. However the scalar field redefinition has to preserve the vacuum of the theory. Besides, the gauge couplings and masses depend on the vacuum expectation value of ϕ and are dynamically determined. This means that Standard Model and Einstein Gravity (in the conformal-affine sense [3]) could be recovered *without the hierarchy problem.* As discussed in [10], it is possible to show that the operators generated by the self-interaction of the scalar field are of the form

$$\frac{1}{M_\sharp^{N-4}}\lambda^{\frac{N}{2}}\phi^N \tag{19}$$

and they are always suppressed by the small parameter λ and do not destabilize the potential of the theory. This result holds also for perturbative corrections coming from quantum gravity.

Considering again the problem of mass generation, one can assume that particles of Standard Model have sizes related to the cut off, that is $M_\sharp^{-1}$, and their collisions could lead to the formation of bound states as in [7]. Potentially, such a phenomenon could mimic the decay of semi-classical quantum black holes and, at lower energies, it could be useful to investigate substructures of Standard Model. This means that we should expect some strong scattering effects in the TeV region involving the coupling of ϕ to the Standard Model fields. The "signature" of this phenomenon could lead to polarization effects of the particle beam. Furthermore the strong dynamics derived from the phenomenon could resemble compositeness as discussed in [11]. Furthermore, bounds on the production of mini-black holes can be derived from astroparticle physics. In [12] a bound on the cross-section is

$$\sigma_{\nu N\rightarrow BH+X} < \frac{0.5}{\text{TeV}^2}. \tag{20}$$

Assuming, in our case, the cross-section $\sigma = M_\sharp^{-2}$, we get a bound of TeV order. If the fundamental scale of our theory is of this order, strong scattering processes at LHC would have the cross-section

$$\sigma_{(pp\rightarrow grav.ghosts+X)} \sim 1\times 10^7 \text{fb} \tag{21}$$

and would dominate the cross-sections expected from the Standard Model. In this case, the Higgs boson could not be detected and no hierarchy problem would be present.

In summary, Higgs mechanism is an approach that allows: *i*) to generate the masses of electroweak gauge bosons; *ii*) to preserve the perturbative unitarity of the S-matrix; *iii*) to preserve the renormalizability of the theory.

The masses of the electroweak bosons can be written in a gauge invariant form using either the non-linear sigma model [14] or a gauge invariant formulation of the electroweak bosons. However if there is no propagating Higgs boson, quantum field amplitudes describing modes of the electroweak bosons grow too fast violating the unitarity around TeV scales [16]. There are several ways in which unitarity could be restored but the Standard Model without a Higgs boson is non-renormalizable at perturbative level.

A possibility is that the weak interactions become strongly coupled at TeV scales and then the related gauge theory becomes unitary at non-perturbative level. Another possibility for models without a Higgs boson consists in introducing weakly coupled new particles to delay the unitarity problem into the multi TeV regime where the UV limit of the Standard Model is expected to become relevant. In [17], it is proposed that, as black holes in gravitational scattering, classical objects could form in the scattering of longitudinal W-bosons leading to unitary scattering amplitude.

These ideas are very intriguing and show several features of electroweak interactions. First of all, the Higgs mechanism is strictly necessary to generate masses for the electroweak bosons. Beside, some mechanisms can be unitary but not renormalizable or vice-versa. In summary, the paradigm is that three different criteria should be fulfilled: *i*) a gauge invariant generation of masses of electroweak bosons, *ii*) perturbative unitarity; *iii*) renormalizability of the theory.

Here we have proposed an alternative approach, based on Extended Theories of Gravity deduced from a 5D-manifold, where the Standard Model is fully recovered enlarging the gravitational sector but avoiding the Higgs boson and the hierarchy problem.

It is important to point out that, in both the non-linear sigma model and in gauge invariant formulation of Standard Model, it is possible to define an action in terms of an expansion in the scale of the electroweak interactions v. The action can be written as

$$\mathcal{A} = \mathcal{A}_{SMw/oHiggs} + \int d^4x \sum_i \frac{C_i}{v^N} O_i^{4+N}\,, \tag{22}$$

where O_i^{4+N} are operators compatible with the symmetries of the model. The electroweak bosons are gauge invariant fields defined by

$$\underline{W}^i_\mu \;=\; \frac{i}{2g}\mathrm{Tr}\;\Omega^\dagger \overleftrightarrow{D}_\mu\,\Omega\tau^i\,, \tag{23}$$

with $D_\mu = \partial_\mu - igB_\mu(x)$ and

$$\Omega = \frac{1}{\sqrt{\phi^\dagger\phi}}\begin{pmatrix} \phi_2^* & \phi_1 \\ -\phi_1^* & \phi_2 \end{pmatrix}\,, \tag{24}$$

where

$$\phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}. \tag{25}$$

is a $SU(2)_L$ doublet scalar field which is considered to be a dressing field and does not need to propagate. The same approach can be applied to fermions [19].

The analogy between the effective action for the electroweak interactions (22) and that of Extended Gravity is striking. Considering only the leading terms, the above theory can be written as a Taylor series of the form

$$f(R) \simeq \Lambda + f_0' R + \frac{1}{2!}f_0'' R^2 + \frac{1}{3!}f_0''' R^3 + ... \tag{26}$$

where the coefficients are the derivatives of $f(R)$ calculated at a certain value of R. Clearly, the extra gravitational degrees of freedom can be suitably transformed into scalar fields ϕ which allows to avoid the hierarchy problem [3]. Both electroweak theory and Extended Gravity have a dimensional energy scale which defines the strength of the interactions. The Planck mass sets the strength of gravitational interactions while the weak scale λ determines the range and the strength of the electroweak interactions. As shown above, these scales can be compared at TeV energies.

In other words, the electroweak bosons are not gauge bosons in standard sense but they can be "derived" from the further gravitational degrees of freedom emerging in Extended Gravity. The local $SU(2)_L$ gauge symmetry is imposed at the level of quantum fields. However there is a residual global $SU(2)$ symmetry, *i.e.* the "custodial symmetry". In the case of gravitational theories formulated as the $GL(4)$-group of diffeomorphisms, tetrads are an unavoidable feature necessary to construct the theory. They are gauge fields which transform under the local Lorentz group $SO(3,1)$ and under general coordinate transformations, the metric $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$ which is the field that is being quantized, transforms under general coordinate transformations which is the equivalent of the global $SU(2)$ symmetry for the weak interactions (in our case the residual $GL(2) \supset SU(2)$) . Such an analogy between the tetrad fields and the Higgs field is extremely relevant. We can say that the Higgs field has the same role of the tetrads for the electroweak interactions while the electroweak bosons have the same role of the metric.

A gravitational action like (26) is, in principle, non-perturbatively renormalizable if, as shown by Weinberg, there is a non-trivial fixed point which makes the gravity asymptotically free [24]. This scenario implies that only a finite number of the Wilson coefficients in the effective action would need to be measured and the theory would thus be predictive and probed at LHC.

Measuring the strength of the electroweak interactions in the electroweak W-boson scattering could easily reveal a non-trivial running of the electroweak scale v. If an electroweak fixed point exists, an increase in the strength of the electroweak interactions could be found, as in the strongly interacting W-bosons scenario, before the electroweak interactions become very weak and eventually irrelevant in the fixed point regime. In analogy to the non-perturbative running of the non-perturbative Planck mass, it is possible to introduce an effective weak scale

$$v^2_{eff} = v^2 \left(1 + \frac{\omega}{8\pi}\frac{\mu^2}{v^2}\right) , \tag{27}$$

where μ is an arbitrary mass scale, ω a non-perturbative parameter which determines the running of the effective weak scale and v is the weak scale measured at low energies. If ω is positive, the electroweak interactions would become weaker with increasing center of mass energy. This asymptotically free weak interaction would be renormalizable at the non-perturbative level without having a propagating Higgs boson again in analogy to Extended Gravity [9].

The asymptotically free weak interaction scenario could also solve the unitarity problem of the Standard Model without a Higgs boson. In this case, there are five amplitudes contributing at tree-level to the scattering of two longitudinally polarized electroweak W-bosons. Summing these five amplitudes, one finds at order s/M_W^2

$$\mathcal{A}(W_L^+ + W_L^- \to W_L^+ + W_L^-) = \frac{s}{v^2_{eff}}\left(\frac{1}{2} + \frac{1}{2}\cos\theta\right) , \tag{28}$$

where s is the center of mass energy squared and θ is the scattering angle. Clearly if v_{eff} grows fast enough with energy, the ultra-violet behaviour of these amplitudes can be compensated and the summed amplitude can remain below the unitary bound. A similar proposal has been made to solve problems with unitarity in extra-dimensional models [20].

It is important to stress that our approach does not require new physics but takes only into account the whole budget of gravitational degrees of freedom. The monitoring of the strength of the electroweak interactions in the W-bosons scattering at LHC could establish the existence of a fixed-point in the weak interactions. Using the one-loop renormalization group of the weak scale could help in formalizing this picture [21]. To be more precise, let us consider the scale of electroweak interactions

$$v(\mu) = v_0 \left(\frac{\mu}{\mu_0}\right)^{\frac{\gamma}{16\pi^2}} , \tag{29}$$

where

$$\gamma = \frac{9}{4}\left(\frac{1}{5}g_1^2 + g_2^2\right) - Y_2(S) \tag{30}$$

and

$$Y_2(S) = \mathrm{Tr}(3\mathrm{Y}_u^\dagger \mathrm{Y}_u + 3\mathrm{Y}_d^\dagger \mathrm{Y}_d + \mathrm{Y}_e^\dagger \mathrm{Y}_e) , \tag{31}$$

where Y_i are the respective Yukawa matrices. If the theory is in the perturbative regime e.g. at m_W, the Yukawa coupling of the top dominates since at this scale $g_1 = 0.31$ and $g_2 = 0.65$ and γ is negative. In this case, the scale of the weak interactions become smaller. If the weak interactions become strongly coupled at TeV region, g_2 becomes large and γ is expected to become positive. We obtain the expected running and the weak scale becomes larger. This is not possible in the framework of a perturbative approach. This result could represent a "signature" for the approach presented here. However, we stress once again that there are indications of a non-trivial fixed point for the non-linear sigma model using exact renormalization group techniques [22]. In conclusion, the unitarity problem of the weak interactions could be fixed by a non-trivial fixed point in the renormalization group of the weak scale. A similar mechanism could also fix the unitarity problem for fermions masses [23] if their masses are not generated by the standard Higgs mechanism but in the same way considered here (let us remind that also $SU(3)$ could be generated by the splitting of $GL(4)$-group). In the case of electroweak interactions this approach could be soon checked at LHC but good indications are also available for QCD [25].

4. Discussion and conclusions

The goal of the present approach is to pursue a unification scheme of fundamental interactions based on *i*) a non-perturbative dynamics, *ii*) the non-introduction of ad hoc hypotheses and *iii*) the consideration of the minimal necessary number of free parameters and dimensions. In principle, different Extended Theories of Gravity can be conformally related each other and derived from a 5D manifold where the fifth dimension can assume the meaning of "mass generator". In other words, it is possible to derive a unification scheme based on the assumption that a 5D-space can be defined where conservation laws are always and absolutely conserved. Such a General Conservation Principle [15] holds since we ask for the validity of the 5D-Bianchi identities which must be always non-singular and invariant for every diffeomorphism. The 5D-space is a smooth, connected and compact manifold where we can derive field equations, geodesic equations and a globally defined Lorentz structure. The standard physics emerges as soon as we reduce from 5D to 4D-space recovering the $GL(4)$-group of diffeomorphisms. By the reduction procedure one is capable of generating the masses of particles and their organization in families. The byproduct is a 4D effective theory of gravity where further gravitational degrees of freedom naturally emerge, induced by the fifth dimension. In other words, we do not recover the Standard GR but Extended Theories of Gravity where non-minimal coupling, scalar field self-interaction potentials and higher-order curvature terms have to be considered. These theories can be confronted and related by conformal transformations.

Furthermore, the $GL(4)$ group of diffeomorphisms can be suitably split generating the fundamental groups of physical interactions. In this respect, a possible the group splitting can be

$$\underbrace{\underbrace{GL(4)}_{4\times 4}}_{\text{diffeom.}} \supset \underbrace{\underbrace{SU(3)}_{3^2-1}}_{\text{gluons}} \otimes \underbrace{\underbrace{SU(2)}_{2^2-1}}_{\text{vec. bosons}} \otimes \underbrace{\underbrace{U(1)}_{1}}_{\text{photon}} \otimes \underbrace{\underbrace{GL(2)}_{2\times 2}}_{\text{gravitons}} \tag{32}$$

with further gravitational degrees of freedom [9].

The main feature of this approach is that higher-order terms or induced scalar fields enlarge the gravitational sector giving rise to massless, massive spin-2 gravitons and massive spin-0 gravitons [3, 29]. Such gravitational modes results in 6 polarizations, according to the prescription of the Riemann theorem stating that in a given N-dimensional space, $N(N-1)/2$ degrees of freedom are possible. The massive spin-2 gravitational states are ghost particles. Their role result relevant as soon as we can define a cut-off mass at TeV scale (the vacuum state of the scalar field) that allows both to circumvent the hierarchy problem and the detection of the Higgs boson. In such a case, the Standard Model of particles should be confirmed without recurring to perturbative, renormalizable schemes involving new particles. The weakness of self-interaction coupling would guarantee the fact that gravity could be compared, at TeV scale, with electroweak interaction.

However, some crucial points have to be considered in order to improve of the proposed approach. The main goal of our scenario is that the Standard Model of particles could be generated by the effective gravitational interactions coming from higher dimensions. In particular the gauge symmetries and mass generations could be achieved starting from conservation laws in 5D. It is important to stress that the Standard Model does not mean only the gauge interaction but also quarks and leptons with their mass matrices that have to be exactly addressed. In particular, the fermion sector has to be recovered.

It is well-known that the standard gauge interactions contains the chiral gauge interactions, which, in our picture, have to be generated from the gravitational interactions otherwise there is no possibility to distinguish between the left-handed and the right-handed particles. In particular, the $SU(2)$ part of the standard gauge interactions, generated from $GL(4)$, has to be chiral and, consequently, fermions acquire a chiral representation. To this end, torsion fields have to be incorporated for the following reasons. As discussed in [13], the Cartan torsion tensor plays the role of spin source in the gravitational field equations where the affine connection is not simply Levi-Civita. Furthermore, as demonstrated in [2], torsion plays an important role in Extended Theories of Gravity since brings further gravitational degrees of freedom responsible of chiral interactions. In other words, torsion is not only the source of spin but, thanks to the non-trivial structure of connections $\Gamma^{\alpha}_{\beta\gamma}$, can give rise to chiral interactions of geometric origin [26]. This means that the $SU(2)_L$ and $SU(3)$ could be related to a gravitationally induced symmetry breaking process where torsion plays a fundamental role. Due to these facts, the present approach has to be generalized including torsion fields. Phenomenological studies considering torsion and fermion interactions are already reported in [27].

Furthermore, as shown in [26], space-like, time-like and null torsion tensors, generated by non-trivial combinations of vector and bi-vector fields, can be classified and represented by matrices which could explain mass matrices of fermions and the hierarchy in the generation of quarks. This approach agrees with other approaches where the effects of gluonic condensates in holographic QCD can be encoded in suitable deformations of 5D metrics (see e.g. [18]).

A detailed study in this sense will be the argument of forthcoming studies.

The validity of the presented scheme could be reasonably checked at LHC in short time, due to the increasing luminosities of the set up. In fact, the LHC experiments (in particular ATLAS and CMS) are indicating, very preliminary, the presence of resonances and condensate states that confirm the Standard Model but, up to now, cannot be considered as evidences for the Higgs boson [25]. Similar results, but with larger integrated luminosity, are reported also by the CDF collaboration at Fermi Lab [28]. The interpretation of such data could be that the further gravitational modes discussed here would induce the formation of resonances and condensates giving rise to a sort of gravitational Higgs mechanism.

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