PURUIT AND EVASION WITH TEMPORAL NON-LOCALITY AND STOCHASTICITY

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Abstract
We discuss a new aspect of an old mathematical problem of chase and escape. We consider one group chases another, called ``group chase and escape", by presenting simple models. We have found that even a simple model can exhibit rich and complex behavior. The model has been extended to investigate the effects of (a) stochasticity in chasing and escaping movements, (b) reaction delays (temporal non-locality) when chasing, and (3) the conversion of caught escapees to new chasers. We show that these effects can add further complexity and result in unexpected behaviors.

1. Introduction
"Pursuit and Evasion" (or " Chases and Escapes ") is a traditional mathematical problem (Nahin 2007). Typical questions include "How much time is needed for a chaser to catch a target?" and "What is the best escaping strategy?" There has been much mathematical interest in obtaining analytical results, so the majority of the questions have dealt with cases in which one chaser is pursuing a single escapee.We recently proposed a simple extended model called "Group Chase and Escape" (Kamimura 2010a) in which one group chases another group. This extension connects the traditional problem of "Chases and Escapes" with current interest in the collective motions of self-driven particles such as animals, insects, cars, etc (Chowdhury 2000, Helbing 2001, Vicsek 2010a).

In this paper, we briefly present our basic model and its rather complex behaviors. Each chaser approaches its nearest escapee while each escapee moves away from its nearest chaser. Although there is no communications within groups, aggregate formations are observed both for both chasers and escapees. How these behaviors appear as a function of parameters, such as densities will be discussed.

In addition, we have extended our models in three main ways. First, we introduced a stochasticity. Players now make errors in which direction they step with some probability. It turns out that some levels of fluctuations work better for more effective capturing. Second, we introduced a temporal non-locality in the form of a reaction delay in a chaser who is pursuing an escapee that is moving with a uniform speed in a circular path. We did not observe a complex chaser's trajectory with constant reaction delay, but distance--dependent reaction delays can cause quite complex behaviors. Finally, we report briefly on the effect of the probabilistic conversion of the captured escapees into new chasers.

2. Basic model
Here, we describe our basic "Group Chase and Escape" model (Kamimura 2010a). Essentially, it is a chase and escape problem in which one group chases another. In order to keep our extension simple, we made each chaser in a chasing group take one step toward its nearest escapee, while each escapee takes one step away from its nearest chaser. They do this independently of each other, meaning there is no communication or direct interaction among members within either the chasing or the escaping groups. We also decreed that a caught escapees be removed from the field, so that gradually the number of escapees decreases. The chase and escape finishes when all the escapees are caught and removed from the field ("complete capture").

There are various possible implementations of this conceptual model. To start with, we considered a square lattice with a periodic boundary condition and discrete step and time movements of the players. We also introduced an exclusion volume property: they cannot move if another of the same type (chaser or escapee) occupies the next location of intended motion. Also, when there are multiple choices (typically only two, due to the square lattice) for the next step, one of them is chosen with equal probability.

We have simulated the above model under various conditions (Kamimura 2010a, Matsumoto 2011). One of the interesting qualitative behaviors observed is a formation of aggregates by both chasers and escapees in spite of the fact that there is no direct interaction among the members of each group. In a related matter, given the initial size of the number of escapees, there exists an optimal number of chasers for effective capture. This can be seen by increasing the number of chasers with a given number of escapee, and noting the time taken to finish capturing all of the escapees. This complete capture time will decrease at a rather fast pace until it reaches the optimal number of chasers, after which it changes at a much slower rate. One of the reasons for this is the excluding volume effect (mentioned above): chasers get in each other's way. However, this is not the only cause -- the very act of chasing and escaping is also a crucial factor, as such an effect is not seen if we set both chasers and escapees as groups of random walkers.

3. Extended Model

In this section, we discuss extension of the basic model to include stochasticity (fluctuations), temporal non-locality (delay), and conversions.

3.1. Effects of Stochasticity

We have extended our study to examine the effects of fluctuation in the above basic model (Kamimura 2010a; 2010b; 2011, Matsumoto 2010). Specifically, we introduced errors in taking steps by players of both sides. With some probability, a chaser now takes a step in the wrong direction, thus increasing its distance from the nearest escapee. This error probability, which is also introduced in the steps of the escapees, is designed so that with the maximal error both sides become groups of random walkers, while with zero errors the model is reduced to the basic model described in the previous section.

We simulated the model with the above fluctuation error with varying probabilities and different ratios of the numbers of chasers and escapees. Increasing the error rate naturally led to a longer time for complete capture, and this is what happened when the number of chasers was relatively large. However, a rather interesting situation was observed when there were small number of chasers and escapees compared to the size of the square grid field. In this case, there exists the optimal level of fluctuation with which the time for complete capture became minimal -- indeed less than not only the case in which both sides were randomly walking, but also in the case of the basic model mentioned above.

Cases that exhibit an appropriate level of fluctuation leading to "better" effects are being studied in various fields, including biology, material science, engineering and so on, under the name of "stochastic resonance" (Wiesenfeld 1995, Bulsara 1996, Gammaitoni 1998). Our observation here can be considered as one of such examples.

3.2. Effects of Temporal Non-Locality

Next, we consider the effects of temporal non-locality in the form of delayed reaction time on the part of the chasers (Ohira 2011, Milton 2011). To examine this, we go back to one of the original one-to-one chase and escape problems in which the escapee moves in a circular path at a constant speed while the chaser moves with its velocity vector pointing to the current position of the escapee. We know that if the speed of the chaser is not as fast as the escapee, the capture is not possible, and the path of the chaser will approach to a "limit circle". The center of the two circles is the same and the ratio of the radii is the same as the ratio of the speeds of the chaser and the escapee.

Now let us consider a case in which the speeds of the chaser and escapee are the same. In this case, the chaser moves behind the escapee with the same uniform distance on the same circle. We now introduce a delay to the reaction time of the chaser. Specifically, the velocity vector of the chaser points to the past position of the escapee. We can immediately see that if we introduce this reaction time as fixed constant (fixed delay), the qualitative behavior of the chaser's motion does not change: the effect is merely an increase of the distance between the chaser and the escapee on the same circle.

However, if we set the reaction time to be proportional to the distance between the chaser and the escapee (distant dependent delay: the longer the distance, the longer the delay), the path of the chaser deviates from the circle. As we increase the rate of this proportionality, the path will become quite complex.

Delay--induced complex behaviors have been studied in various contexts. The most notable examples are the dynamical trajectories given by delay differential equations, such as the MacKay -- Glass model (Mackey 1977). In this model, a very simple first order differential equation with an external force term, which is a function of the delayed past state of the dynamical variable, can show various dynamics from a stable fixed point, limit cycles, and further create complex chaotic trajectories as we increase the value of the delay. Our observation here can be considered another example of delay--induced complexities.

3.3. Effects of Conversion

Finally, we briefly discuss the effect of conversions (Nishi 2012). We extend the basic model in such a way that the caught escapee becomes a new chaser with a certain probability, while escapees can proliferate with some probability as well. The balance between these two factors again produces a non-monotonic change in the time it takes for complete capture with varying parameters. For example, if we fix the number of chasers and escapees and change the proliferation probability, there exists an optimal value to have the longest time for complete capture for a smaller value range of conversion probabilities. This finding has also been reported in a separate publication (Nishi 2012).

Using our model with this extension has potential for application to studies on the spread of certain epidemics such as rabies.

4. Discussion

We have described our recent proposal and investigation of group--based chases and escapes and are faced with the following tasks. First, we should make our model more realistically by including communication within groups or more complex chasing and escaping strategies. They reflect such cases of one group of animals chasing another, e.g., wolves hunting deer (Vicsek 2010b). Second, we need to consider a possible application to distributed robotics or other engineering systems. For example, the human immune system includes neutrophil granulocytes that chase foreign external viruses or chemicals. Could we implement such a defense system against attacks in cyberspace by adapting the concept of chase and escape included? Finally, this type of chase and escape interactions among the constituents in groups has not been addressed in studies of physical theories. Extension of many-body theories of statistical physics for the purpose of adapting them to chase and escape--type interactions will likely pose interesting challenges in the future.

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