ARE ANOMALOUS COSMIC FLOWS A CHALLENGE FOR LCDM?

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Abstract
The dipolar moment of the peculiar velocity field, named bulk flow, is a sensitive cosmological probe: in parallel with the estimation issued from density fluctuations, it can give an indication on the value of cosmological parameters. Recent observations, based independently on composite velocity survey or velocity reconstruction from redshift survey, showed the existence of an anomalously high bulk flow in apparent contradiction with the linear prediction in ΛCDM cosmology. Using numerical simulations, we interpret this observation of consistently large cosmic flows on large scales as a signature of a rare event. Supposing we live in such a configuration and building samples with bulk flow profiles in agreement with the observations, we show that the asymmetric distribution of matter of large scales is responsible for the observed high bulk flow. To confirm the possible origin in ΛCDM cosmology of such a bulk flow profile and the agreement with linear theory description, we carefully study the time-dependence of the bulk flow and the distribution of matter.

1. Introduction
Recent observations such as velocity surveys (Watkins 2009) and redshift surveys (Erdogdu 2006) showed the existence of an excess in the dipolar moment of the velocity fields (i.e. bulk flow) at scales up to 50 h^{-1} Mpc. New observations, done through kinetic Sunyaev-Zeldovich effect, show this deviation can even be higher at larger scale (Kashlinsky 2009). Such discordance can be interpreted only in two ways: either the cosmological model we assume to predict the bulk flow is wrong, either we live in an environment with a very unlikely bulk flow profile. Several authors (e.g. Feldman 2010) favorize the cosmology as the cause of such a discrepancy.
On the other hand, the question of the realization of rare events is crucial in many fields in physics. Due to our unique position of observers, it is of utmost importance in cosmology to understand the behaviour of a given observable.
In this proceeding, we interpret the observation of the anomalously high bulk flow as a rare event realization. In particular, using high-resolution N-body simulations, we show that the bulk flow can be largely out of the linear prediction even though all the dynamics remain linear. To highlight the linearity of the dynamics, we draw a link between the velocity fields and the asymmetric distribution of matter and study the independence of this link on time evolution.

2. Realization of rare events
The velocity of matter in the Universe **u**(**r**) can be decomposed into the sum of the mean Hubble expansion $\mathbf{v}_H = H_0\,\mathbf{r}$ and a field of velocity fluctuations $\mathbf{v}(\mathbf{r}) = \mathbf{u}(\mathbf{r}) - \mathbf{v}_H$. This field is called the peculiar velocity field.
The bulk (i.e. volume average) flow is defined as the mean of **v**(**r**) in a sphere of growing radius surrounding the observer. Assuming that each component of the peculiar velocity field is isotropic and follow a Gaussian distribution, the Fourier components are uncorrelated. Under those assumptions, the velocity power spectrum (convolved by a top-hat window function in Fourier space) describes entirely the statistical behaviour of the field (Gramann 1998):

$$v_{bulk}(R) = \sqrt{\frac{1}{2\pi^2}\int_0^\infty k^2 P_v(k)\hat{W}(kR)^2\,dk}\ . \qquad (1)$$

Physically, we understand easily that the matter distribution source velocity fields. Especially, in linear theory, the velocity power spectrum and the density power spectrum are linked in a simple way:

$$P_v(k,z) = \frac{H^2(z) f^2(z)}{k^2} P_\delta(k,z)\,, \qquad (2)$$

where H is the Hubble factor and $f = \frac{d\ln\delta}{d\ln a}$ is the linear growth rate of density fluctuations. This relation is crucial to describe the time-dependence of the bulk flow since it relates the velocity fluctuations to the density fluctuations, which are evolving according to the linear growth rate D+ of a given cosmological model. Therefore, the time-dependence of the bulk flow can be written:

$$v_{bulk}(R,z) = \left\{ \frac{H(z) f(z) D_+(z)}{H(z=0) f(z=0) D_+(z=0)} \right\} v_{bulk}(R, z=0)\,. \qquad (3)$$

We are deeply interested in predicting the probability of obtaining the observed bulk flow profile in a given cosmology. For simplicity, we approximate the observational profile (Watkins 2009) by only two data points, namely a depletion point at radius R=16 h^{-1} Mpc and a bump at radius R=53 h^{-1} Mpc (see Figure 1).
The probability of having a given value for the norm of the bulk flow is obviously not gaussian. In complete analogy with a gas of particles, the probability distribution of the norm of the velocity vector is a maxwellian. Extending this formalism to the case of the joint probability of having fixed velocities at radius R1 and R2 and following (Lavaux 2010), we obtain a two-dimensional maxwellian distribution:

$$P(v_{R1}, v_{R2}) \propto \frac{2}{\pi} \frac{v_{R1}^2 v_{R2}^2}{\left|\det M\right|^{3/2}} \times \exp\left(-\frac{1}{2} {}^t\vec{V} M^{-1} \vec{V} \right) \text{ with } \vec{V} = \left(\|\vec{v}_{R1}\|, \|\vec{v}_{R2}\| \right) \qquad (4)$$

Since the bulk flow is computed in a spherical volume, the correlation matrix M cannot be diagonal. The velocities at scale R2 are strongly dependent on the velocities at scale R1. As a result the correlation terms can be expressed as:

$$\gamma_{ij} = \sqrt{\frac{1}{2\pi^2} \int_0^\infty k^2 P_v(k) \hat{W}(kR_i) \hat{W}(kR_j)\, dk}\,. \qquad (5)$$

Computing those quantities, we find that the probabilities of having such a bulk flow, given by the long tail of the maxwellian, is very scarce: only 1.4% in ΛCDM.

3. Bulk flow and asymmetric distribution of matter

3.1. Numerical set-up

We have performed a large set of high performance N-body simulations of large-scale structures with various dark energy components.
For this proceeding, we consider a simulation done within a cubic region of 648 h^{-1} Mpc with 1024^3 particles. Thanks to an adaptive mesh refinement method, the maximum resolution in the ΛCDM cosmology reaches 10 h^{-1} kpc.
The bulk flow is computed in a similar way to the definition given in section (2): from the tridimensional velocity fields of N_h objects in a sphere of radius R, we have:

$$v_{bulk}(R) = \left\| \frac{1}{N_{h,r<R}} \sum_i^{N_{h,r<R}} \vec{v}_i \right\| \qquad (6)$$

We throw randomly 20.000 centres in the simulation volume, and around each center, we compute the bulk flow profile. From the whole environment, we could extract two interesting subsets (shown in Figure 1):

- Set of centres with a bulk flow profile close to the mean statistical prediction given in equation (1) at 95% confidence level: this is the *linear* catalogue.

- Set of centres with a numerical bulk flow close to the bulk flow profile measured by (Watkins 2009) at 95% confidence level. Since the mean bulk flow of this sample is on the mean in agreement with the observations, we call this subset *realistic*.

The probability to find a bulk flow profile in ΛCDM cosmology is given by the ratio of the number of elements in the *realistic* catalogue by the overall number of centres. The analysis of the Grand Challenge simulation exhibits 255 centres over 20.000 in the *realistic* catalogue. The numerical probability is then 1.27%.

Considering the fact that this experimental probability is computed on all datapoints whereas the theoretical probability is computed from two points only, we have a good agreement between both the numerical and the statistical views. The Watkins-like profile can then be interpreted like a rare event realization.

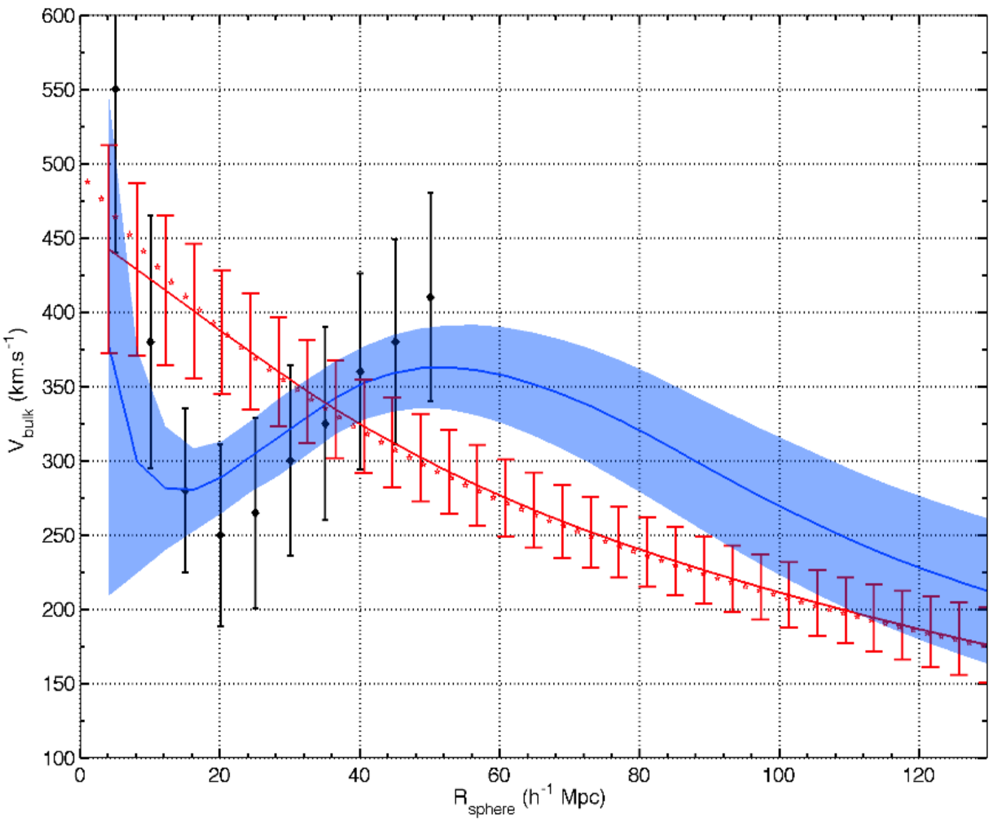

Figure 1: Mean bulk flow profile vs radius of sphere. The observations (Watkins 2009) are indicated with black points; the linear prediction (equation (1)) with red stars; the linear catalogue is in red and the realistic catalogue is in blue.

We see from Figure 1 that there is a strong disagreement for the bulk flow between the *realistic* sample and the linear prediction. Such a bulk flow profile traces possibly an asymmetric matter distribution. Indeed, the bulk flow is a vectorial quantity, which keeps track of local overdensities. Therefore, to understand bulk flow profile, we then have to quantify the local overdensities in a given direction with respect to the other direction.

3.2. Dynamical origin of the bulk flow

How to characterize the asymmetry in a sphere of a growing radius R? Mathematically, the *asymmetry index* is obtained by maximizing the difference of the density fields ρ of the hemispheres at radius R:

$$A_R = \max_{\phi_0 \in [0,2\pi], \theta_0 \in [0,2\pi]} \left\{ \frac{1}{\rho_{mean}} \iint_{S^2/2} \rho_{<R}(\theta+\theta_0, \phi+\phi_0) - \rho_{<R}\left(\pi-(\theta+\theta_0), \pi+(\phi+\phi_0)\right) d\Omega \right\} \qquad (7)$$

The *direction of the asymmetry index* is given by the direction of the north pole of the densest hemisphere. The more the asymmetry index is close to one, the more asymmetric the density field is.
Figure (2) shows the dependence of the asymmetry index on the radius for linear and realistic catalogues. The linear catalogue exhibits a constant decrease with radius whereas two main characteristics are clear on the realistic asymmetric index: from 40 to 76 h^{-1} Mpc, the realistic sample is more symmetric than the linear one; from 76 to 128 h^{-1} Mpc, the realistic catalogue is less symmetric than the linear. Due to the relation between gravity and velocity fields, one can wonder if the excess of the asymmetry around 80 h^{-1} Mpc is the cause of the bump of the bulk flow at 53 h^{-1} Mpc.

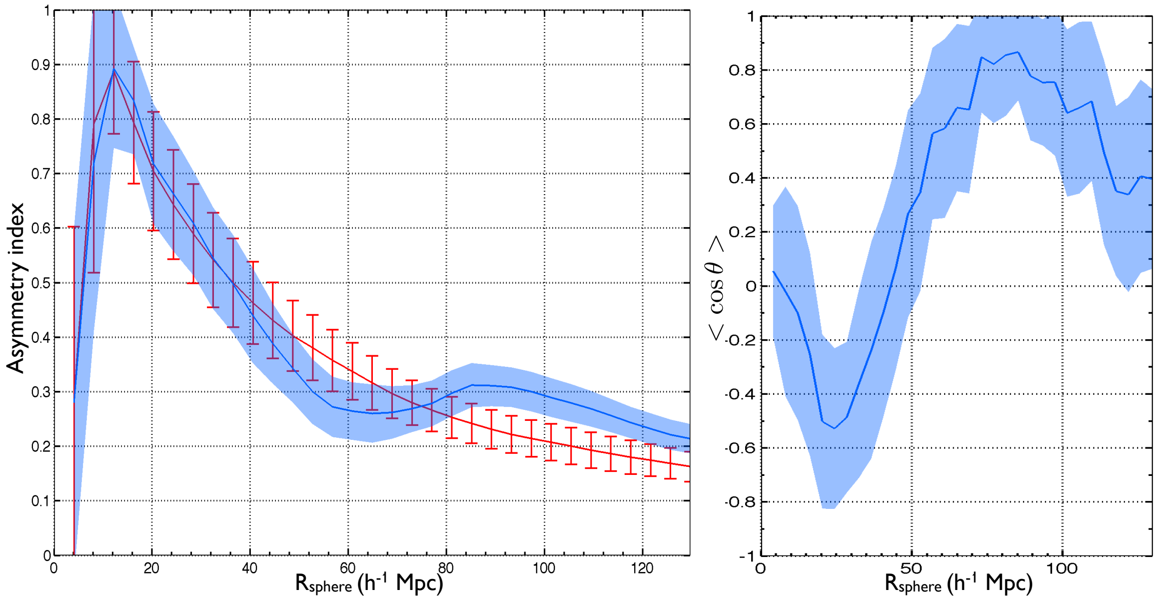

Figure 2: Left panel: Asymmetry index vs radius of the sphere. Linear catalogue is in red line and the realistic sample is in blue. Right panel: Normalized scalar product of the differential direction of the asymmetry index and the bulk flow at 53 h^{-1} Mpc vs radius.

To answer this question, we have to compute the scale of alignment of the bulk flow at the bump position (53 h^{-1} Mpc) with the differential (i.e. in shells) asymmetry index. The right panel of Figure (2) shows this normalized scalar product peaking around a particular scale. Therefore, the sourcing scale of the bulk flow is found to be 85 h^{-1} Mpc. More details as well as a full equivalence with a centre of mass approach can be found in (Bouillot 2012).

4. Bulk flow as a linear quantity

Even if the amplitude of the bulk flow is largely out of the mean linear prediction, this means only that we've to deal with a rare event. The linear nature of the bulk flow is confirmed by its linear time-evolution.
As a matter of fact, we compute the bulk flow and the asymmetry index through time for all objects of the realistic catalogue. Using equation (3), we renormalize bulk flow profiles at various redshifts with the bulk flow profile at z=0. The same renormalization procedure is followed for the asymmetry index:

$$A_R(z) = \frac{D_+(z)}{D_+(z=0)} A_R(z=0)\,. \quad (8)$$

Figure (3) shows clearly that the bulk flow (as well as the asymmetry index) renormalized by the linear evolution, remains the same through time. Since the asymmetry index is build from density contrasts, which is a scalar quantity, it is not surprising that the dynamical evolution is linear.

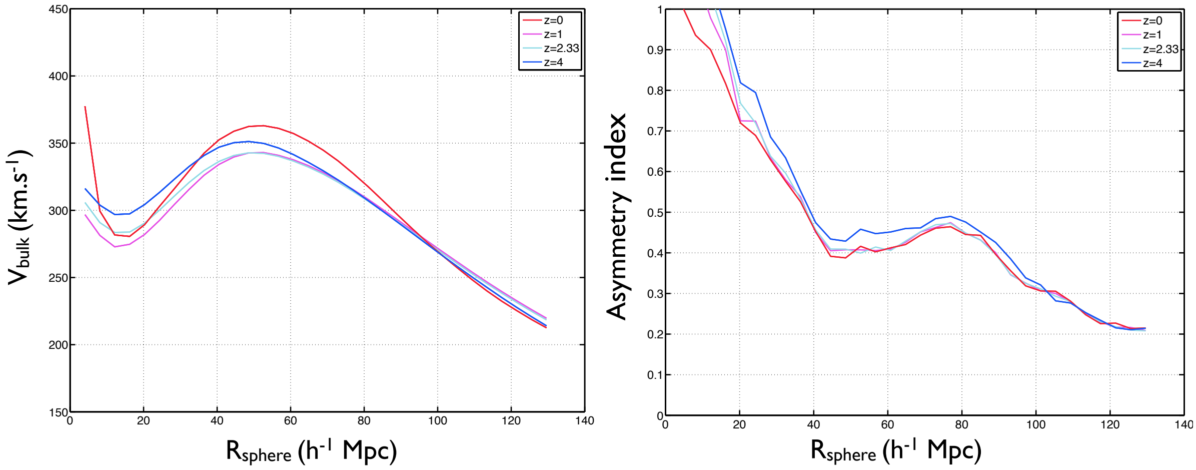

Figure 3: Left panel: Bulk flow at different redshifts normalized to the bulk flow at z=0. Right panel: Asymmetry index at various redshifts normalized to the asymmetry index at z=0.

5. Conclusion

Although the anomalously high amplitude of the bulk flow profile is often interpreted as a challenge for ΛCDM, we show that such a profile can be seen as the imprint of a rare event.
Using numerical simulations, we build samples with bulk flow profiles in agreement with the observations. The probability of having such events is allowed in a ΛCDM model with initial Gaussian condition and is 1.4%, in agreement with the probability issued from our numerical catalogue.
The origin of such a bulk flow profile results from the asymmetric tridimensional distribution of matter. In particular, the bump of the bulk flow at 53 h^{-1} Mpc is explained by the asymmetric distribution of matter at 85 h^{-1} Mpc. Finally, we show that the time-evolution of the bulk flow for such samples is really predicted by the linear theory.
At larger scales (Figure 1), the amplitude converges towards the expected amplitude of the mean linear prediction in ΛCDM. The distance between the bump and this point of reconvergence is a feature of cosmological models (Alimi 2012).

References

Alimi J-M, Bouillot V, Rasera Y (2012) Constraining cosmology from large scale bulk flow MNRAS (in prep)
Bouillot V, Alimi J-M, Füzfa A, Rasera Y (2012) Retrieving cosmological signal using cosmic flows MNRAS (in prep)
Erdogdu P, Lahav O, Huchra J P (2006) Reconstructed density and velocity fields from the 2MASS Redshift Survey MNRAS 373, 45
Feldman H-A, Watkins R, Hudson M J (2010) Cosmic flows on 100 h^{-1} Mpc MNRAS 407,23
Gramann M (1998) Power spectrum of velocity fluctuations in the Universe ApJ 493, 28
Lavaux G, Tully R B, Mohayaee R, Colombi S (2010) Cosmic Flow From Two Micron All-Sky Redshift Survey ApJ 709, 483
Kashlinsky A, Atrio-Barandela F, Ebeling H, Edge A, Kocevski D (2009) A new measurement of the bulk flow of X-ray luminous clusters of galaxies ApJ 712, 81
Watkins R, Feldman H A, Hudson M-J (2009) Consistently large cosmic flows on scales of 100h^{-1} Mpc MNRAS 392, 743