On partially static Kaplan turbines[1]

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Recently it was shown, that it is possible to increase the power of a wind turbine by a factor of 5 by operating it in a partially static mode. In this paper we explore, whether also Kaplan turbines can be made partially static in order to increase their power or in order to operate them under a smaller head. As a result one would hope to decrease the price for hydroelectric energy and to open new applications, which are not accessible to known technology.

Introduction

If a wind turbine operates in ambient air of velocity v_1, and decelerates it to a smaller velocity, v_3, then the velocity at which the wind will pass through the turbine is always going to be $v_2 = 1/2 \cdot (v_1 + v_3)$ [1]. In a classical wind turbine only the flow of air, which passes through the propeller participates at the process of energy extraction, and therefore augmenting v_2 above $1/2 \cdot (v_1 + v_3)$ would result in a violation of energy- and momentum conservation, as was shown in [2].

Augmenting v_2 is possible only if an additional flow of air (passing outside the propeller) participates at the process of energy extraction. This can be achieved by means of a sail- or wing like structure placed in the vicinity of the turbine. It then becomes possible to have the air passing through the turbine at a velocity larger than $1/2 \cdot (v_1 + v_3)$, and even at a velocity larger than v_1. Because in this kind of system the power of the rotating propeller is increased by means of a static wing-like structure, the device was referred to as "partially static turbine" [2]. In the study [2] we reported a power increase of a factor of 5 by means of an external structure around the propeller. A very detailed CFD analysis of a partially static wind turbine can be found in [3]. The thesis [3] also lists some of the past attempts to build a shrouded wind turbine.

One should expect that similar arguments apply to water turbines:

For instance, if water under the pressure $r \cdot g \cdot h$ exits through a straight tube with no turbine present, its maximum velocity will be $v_{\max} = \sqrt{2 \cdot g \cdot h}$ (with: ρ density of water, g gravitational constant, h difference in water level before and after the opening).

If instead the inner surface of the tube is bent like a sail or an airplane wing, then one might expect this bending to create an accelerating force with a component normal to the velocity vector of the fluid and thereby to create a low-pressure field, which could then cause the fluid to achieve speeds higher than $v_{\max} = \sqrt{2 \cdot g \cdot h}$. This increase in velocity could in principle result in an increased turbine power. Or instead one could operate a turbine of given size at a smaller head, getting the same power as from a conventional system at larger head, as will be discussed in more detail.

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We need to take into account however, that energy- and momentum need to be conserved in the overall system: if a unit volume of water has a length (in the direction of the flow) Δr, if it flows through the turbine in a time interval Δt and if the force between this volume and the turbine is F, then the energy transfer is $\Delta E = F \cdot \Delta r$ and the transfer of linear momentum is $\Delta p = F \cdot \Delta t$. Therefore the energy transfer does not depend on the velocity, at which the water flows through the turbine. But the momentum transfer does depend on it. As a consequence only at one particular velocity can both energy- and momentum be conserved in the system.
Therefore, similarly to what we had found for a wind turbine [2], one should expect that also for a flow of water a significant increase of the flow velocity through the turbine can only be achieved if an additional flow, which does not pass through the turbine itself, participates in the process of energy extraction.
In this paper we investigate a very simple version of a Kaplan turbine, which consists of a propeller with the axis of rotation parallel to the flow of the water. The system does not have any distributer and the flow of water does not take any tangential velocity components, apart from the small rotatory movement, which is caused by the action of the rotating propeller itself.
In this paper we only explore, whether partially static turbines can also operate in water, as one would expect. No optimization studies have been performed for this work.

Power of a Kaplan turbine

A Kaplan turbine operates between two water levels with a pressure difference of $r \cdot g \cdot h$, the cross section of the turbine may be S, the turbine is operated with a diffuser. The geometrical configuration is as shown in figure 1. When flowing through the turbine, the water has the velocity v.
We note, that in the absence of any turbine, water would pass at a maximum velocity of $v_{\max} = \sqrt{2 \cdot g \cdot h}$ through the tube, and since the energy density of the water would be $r \cdot g \cdot h$, the flow would have a power of $r \cdot g \cdot h \cdot S \cdot v = S \cdot r \cdot \sqrt{2} \cdot (g \cdot h)^{3/2}$. If instead we extract energy from the flow of water by means of a turbine, the kinematics of the system changes:

In a first step we assume that the pressure drop at the turbine is $r \cdot g \cdot h$ and that the diffuser has a very large opening, such that the velocity of the water at the outlet is approximately zero.
Since before entering the turbine the water has the velocity v, and when leaving the diffuser the velocity 0 (in approximation), the linear momentum density of the water is reduced by

$$\Delta p = r \cdot v \qquad (1)$$

If the walls of the system do not absorb linear momentum (and no energy), this linear momentum must be transferred to the turbine.
The linear momentum transfer to the turbine can also be calculated from the energy density transferred to the turbine, which was assumed to be $\Delta E = r \cdot g \cdot h$, and the velocity, at which this energy transfer takes place,

$$\Delta p = \Delta E / v \qquad (2)$$

Because of momentum conservation, equations (1) and (2) can be put equal, resulting in

$$\Delta p = \Delta E / v = r \cdot g \cdot h / v = r \cdot v \qquad (3)$$

or
$$r \cdot g \cdot h = r \cdot v^2 \quad \Rightarrow v^2 = g \cdot h \qquad (4)$$

Equation (4) means, that the kinetic energy density of the water flow $1/2 \cdot r \cdot v^2$ is only half of the pressure $r \cdot g \cdot h$, therefore with $v_{max} = \sqrt{2 \cdot g \cdot h}$ we get $v^2 = 1/2 \cdot v_{max}{}^2$: for reasons of energy- and momentum conservation the water must flow through the turbine at a velocity which is a factor $\sqrt{2}$ below the maximum possible velocity v_{max}, which we would find in an opening without any turbine present.
Consequently, the power of the turbine would be a factor of $\sqrt{2}$ below the power of the free flow and would result as

$$P = S \cdot r \cdot (g \cdot h)^{3/2} \qquad (5)$$

We next consider, that in reality the water will not leave the diffuser at velocity 0, but at a final velocity *v'*. Because of that, the pressure drop at the turbine gets reduced correspondingly to $r \cdot g \cdot h - 1/2\, r v'^2$ and the difference in linear momentum becomes $r \cdot (v - v')$. We therefore need to modify equation (3) to

$$r \cdot g \cdot h - 1/2\, r \cdot v'^2 = r \cdot (v - v') \cdot v \qquad (6)$$

and therefore $v^2 = g \cdot h - 1/2 \cdot v'^2 + v \cdot v'$
now, v^2 is larger than $g \cdot h$ by the additive term $-1/2 \cdot v'^2 + v \cdot v'$. (This term is larger than 0 for $v > 1/2 \cdot v'$, which is always true, since the velocity of the water at the turbine, v, must necessarily be larger than the velocity at the outlet of the diffuser, v'.)
The power of the turbine now becomes

$$P = F \cdot v = (r \cdot g \cdot h - 1/2 \cdot r \cdot v'^2) \cdot S \cdot \sqrt{g \cdot h - 1/2 \cdot v'^2 + v \cdot v'} \qquad (7)$$

substituting $v = b \cdot v'$, we find that $\frac{dP}{db} > 0$ in the region of interest. This means, that the power of the turbine is maximum for $v' \to 0$. In different words: the larger the outlet of the diffuser, the more power gets available to the turbine, till it reaches the value given in (5).

Above arguments hold only if the flow of water looses its energy only and only when traversing the turbine. There will in general be additional losses, for instance at the walls of the tube. If we assume, that these losses will reduce the energy density of the water by an amount ΔE, and the linear momentum by Δp, then equation (6) becomes :

$r \cdot g \cdot h - \Delta E - 1/2\, r \cdot v^2 = (r \cdot (v - v') - \Delta p) \cdot v$ and correspondingly

$v^2 = g \cdot h - \Delta E / r - 1/2 \cdot v'^2 + v \cdot v' + \Delta p \cdot v$

As a result, v^2 now can become smaller than $g \cdot h$, as had to be expected, since the additional term ΔE has the same effect as lowering the head. Obviously, the power of the turbine will decrease below the value given in (7).

From this discussion we see, that

a) the flow through a simple Kaplan turbine should have a longitudinal velocity which is about a factor of $\sqrt{2}$ below the maximum possible velocity, $v_{\max}$, (eq. 4)
b) it is not convenient to increase this velocity (eq.7), because this does not augment the power and of course
c) it is impossible to augment v above $v_{\max}$ in a conventional Kaplan system.

Fluid dynamic simulation

We use the program Star-CD for simulating a Kaplan type turbine with a radius of 0.56 m (and therefore a cross section of $1m^2$), the Kaplan turbine operates between two large reservoirs of water. As boundary conditions, the walls of the reservoirs are put at 12 kPa and 0 kPa pressure, respectively. In order to simplify the analysis, buoyancy forces are switched off. Only a slice of 10 degrees is simulated, the remaining of the model is taken into account via cyclic conditions on the side walls (the turbine has 36 blades).

We first show in figure 1 a cross section along the axis of rotation (referred to as x-axis) of a Kaplan turbine with a diffuser.

A maximum power of 19 kW is calculated for this turbine, it occurs at a speed of rotation of ω=200 rpm. At this ω, water flows at a mean velocity of 3.4 m/s through the turbine. (For h=1.2 m, $v_{\max} = \sqrt{2 \cdot g \cdot h}$ = 4.9 m/s).

Partially static mode of operation

We next modify the walls of the tube, giving them a bending and moving them outward so that we create a significant flow of water passing between propeller and wall, but not passing through the area, covered by the rotating propeller itself: while the rotor has again a diameter of 0.56 m, the diameter of the narrow section (in which the rotor is mounted) has a diameter of 0.80 m, so that the flow in the narrow section has now a cross section of $2.0m^2$, and only half of it is covered by the rotor itself.

The shape of the wall is from an educated guess, optimization studies have not yet been performed. In figure 2 we show a cross section through the device. Again the same rotor is used as in figure 1 and again the pressure difference between the left and the right reservoir of water is 12 kPa. The figure shows, how the bent surface of the tube is causing a strong lowering of the pressure in the device. One can also observe, that not only the flow through the propeller, but also the additional flow, which does not pass through the turbine itself, does reduce its pressure - both flows are under a pressure of about 12 kPa before entering the partially static turbine system, and both flows are under a pressure of about 0 kPa when leaving the device.

Figure 3 compares the power of the conventional Kaplan turbine (as shown in fig.1) and the power of the partially static turbine (as shown in fig.2) as a function of the speed of rotation. The power of the partially static turbine is increased by about a factor of 3 from 19.0 kW to 56.7 kW, the power maximum has shifted from 200 rpm to 425 rpm.

The velocity of the water flowing through the turbine is now 6.4 m/s. The water, which flows through the narrow section between propeller and inner wall, has a velocity of 7.0 m/s. Since the total flow cross section in the narrow section has doubled compared to the flow though the conventional Kaplan turbine, and since the velocity there has increased in mean by about a factor 1.5, the total

flow through the partially static system is about 3 times larger than the flow though the conventional system.
We next operate the partially static turbine with a pressure difference of 6 kPa instead of 12 kPa, corresponding to a head of only 60 cm. Normally Kaplan turbines are not used with such a small head. We compare in figure 4 the power output of the device under this reduced head of 60 cm with the power of the conventional Kaplan device operating at a head of 1.2 m (as in figure 3). The power output is in both cases about the same.
Note, that in order to get this same power from a head of 60 cm with a conventional Kaplan turbine, the size of the turbine would need to be about 3 times larger than when operating with a head of 1m (provided that the turbine would work at all with such a small difference in water levels).

Conclusion

Operating a Kaplan turbine in a partially static mode, one can increase the velocity of the flow through the turbine. This effect allows for instance, to reduce the size of a turbine of given power and to increase its speed of rotation, reducing in this way the price of construction of the turbine. Or instead one can make use of smaller differences in water levels. This allows to avoid the construction of high dams, with their potentially negative ecologic impact and it opens new perspectives for the use of tidal energy.

References

[1] Betz, A., Wind-Energie und ihre Ausnutzung durch Windmuehlen. (1926), *Oekobuch* reprint, Staufen (1994).
[2] H.Grassmann, F.Bet, M.Ceschia, M.L.Ganis, “On the physics of partially static turbines”, submitted to *Renewable Energy*.
[3] M.L.Ganis, “CFD analysis of the characteristics of a shrouded turbine”, diploma thesis, University of Udine. A copy can be obtained from www.diplom.de.

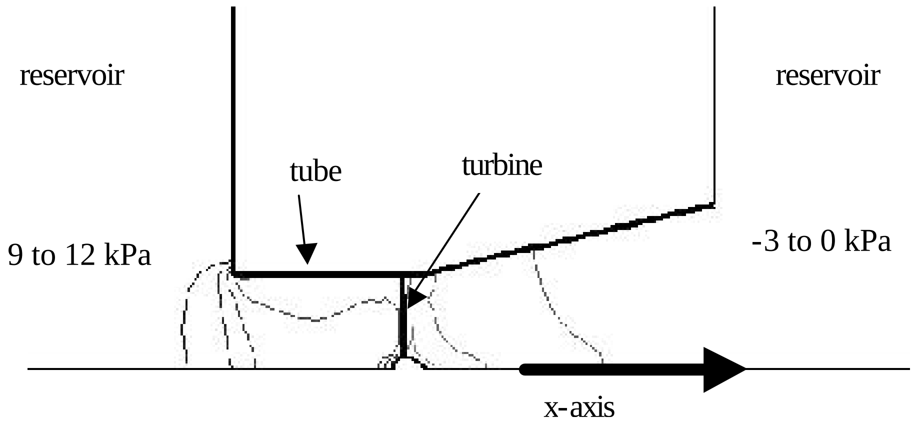

Figure 1: Kaplan-type turbine with diffuser. The scale is in steps of 3 kPa.

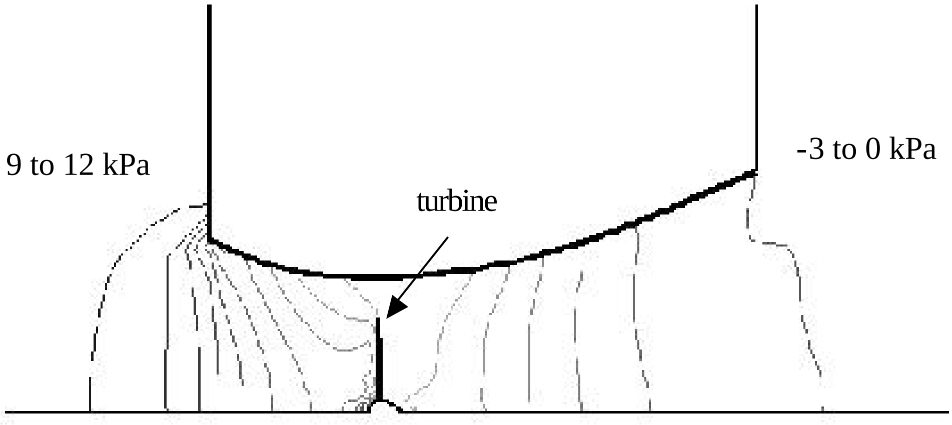

Figure 2: Kaplan-type turbine operated in partially static mode. The pressure scale is identical to the one used in figure 1. One can see how there is a significant distance between the tip of the propeller blades and the inner surface of the wall, leaving room for a flow of water in addition to the flow, which passes through the area, covered by the rotating rotor.

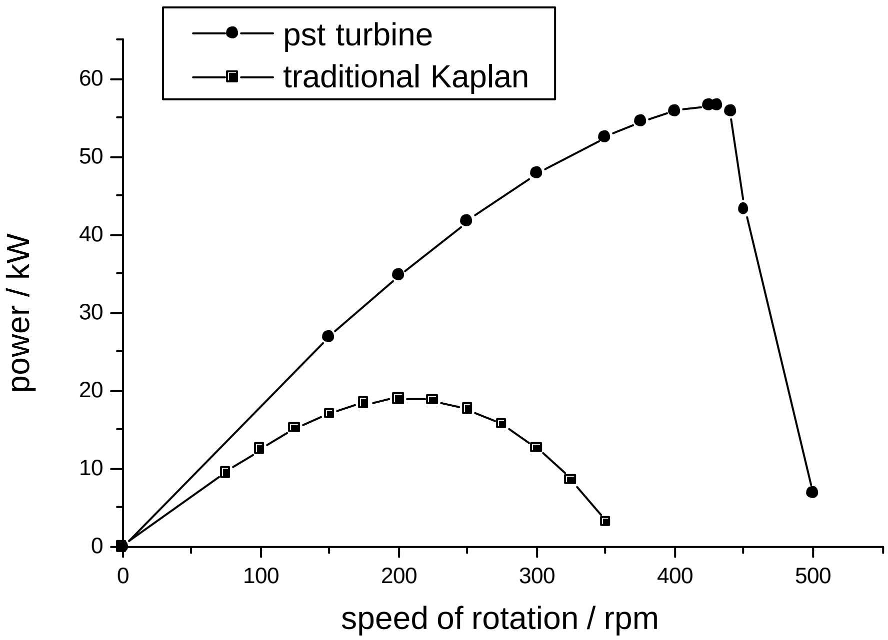

Figure 3: Power of simple Kaplan turbine operated in the traditional mode (as fig.1) compared to the same device operated in a partially static configuration (as fig.2). Both turbines operate under the same pressure difference of 12 kPa.

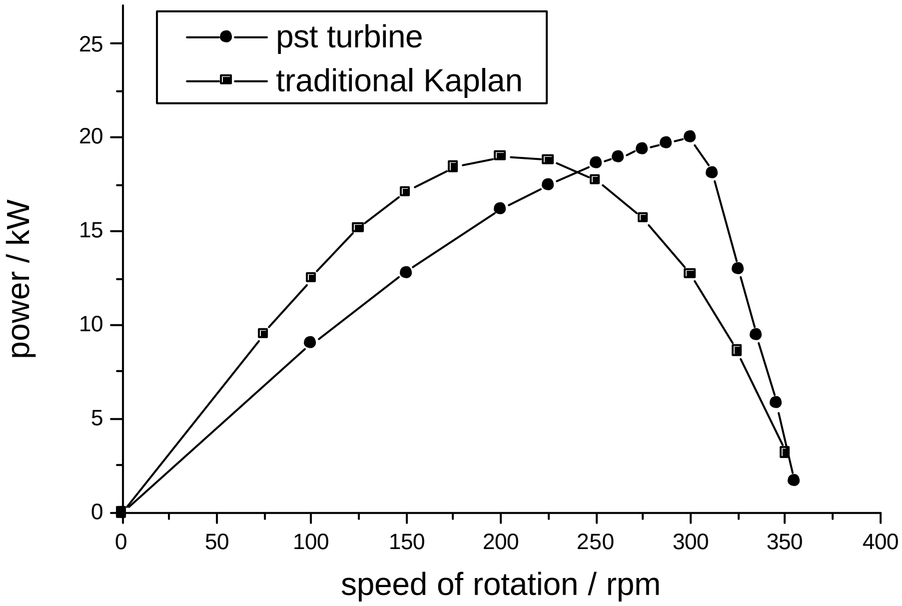

Figure 4: power of conventional Kaplan operating at a head of 1.2 m, compared to power of the same turbine operating as partially static turbine at a head of 60 cm.